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APPLIED MECHANICS REVIEWS

VOL. 4, NO. 1 **MARTIN GOLAND** *Editor* JANUARY 1951

RECENT DEVELOPMENTS IN DYNAMICS AND VIBRATION

J. P. DEN HARTOG

PROFESSOR OF MECHANICAL ENGINEERING, MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASS.

In this review brief mention is made of eleven subjects in vibration which have seen recent activity and on which publications can be expected in the near future. They are listed in what the reviewer believes to be the order of importance from a standpoint of their influence on engineering development.

1 Servomechanisms. The first important paper on this subject dates back to 1877 when Routh treated the problem of engine governors. Publications then followed each other at long intervals until the years just before World War II, when the subject burst into great activity. The principal applications are automatic controls of all sorts for purposes of peace as well as of war. The main change in analytical treatment was a shift from the old Routh stability criterion to the method of Nyquist, in which the response of the system to a steady-state alternating excitation of constant frequency is plotted as a vector in a phase plane. A study of the path of the end point of that vector for all possible frequencies reveals the degree of stability of the system.

Servomechanisms account for a large fraction of all recently published papers on vibration, and undoubtedly this fraction would be greater if all developments on military apparatus were reported in the literature. Almost all current applications of gyroscopes fall in this category.

2 Flutter. Before the war the problem of flutter of wings, tails, and other parts of aircraft was restricted to subsonic speeds. Even now, many papers are being published applicable to subsonic speeds only, and that subject is by no means finished. Flutter calculations still are extremely complicated, although they have been much simplified during the last ten years; further publications in this direction are to be expected in the next few years. While this development was going on, the problem was overtaken by the increase of airplane speeds into the supersonic range, where the flutter characteristics are radically different. Papers on sonic and supersonic flutter currently are numerous and will remain so in the near future.

3 Flames. Almost a century ago when gaslight was a novelty, Tyndall demonstrated and explained the mechanism of the singing gas flame. Recently, this phenomenon has graduated from the toy stage and has acquired great practical importance in gas turbines and jet engines. The large velocity of the gas stream in these devices, as compared with Tyndall's experiment, has reopened the problem of flame stability, which is of considerable complication. Although relatively few publications have been devoted to it recently, it is an active problem in which new developments are to be expected.

4 Nonlinear Vibrations. This subject was started in the 1920's by the Dutchman Van der Pol, and it has now become virtually a Russian monopoly. The number of publications on it in the Russian technical journals (and mentioned in these Reviews) has been large, the greatest name in the field being that of Andronoff. These papers are of considerable interest, because many vibrational systems become nonlinear for large motions, although for most mechanical cases the linear theory is quite adequate. Direct applications of pronounced nonlinearity toward a useful purpose have been made in electrical engineering (saturated magnetic circuits, thermisters, etc.). In mechanical engineering such applications still lie in the future, so that, for the time being, the subject is of secondary importance to mechanical engineers in spite of the numerous current publications.

5 Blades of Gas Turbines and Aircraft Propellers. The calculation of the natural frequencies of the blades of a gas turbine is the same as that for a steam turbine. However, the problem has acquired new importance because of the fact that in the axial-compressor part of a gas turbine it is difficult to obtain nonstalling flow in all the blades, so that stall-flutter excitation is serious.

In aircraft propellers the profiles have become thinner with increased speeds of the aircraft, which has accentuated the vibration problem in them.

6 Shimmy of Wheels. This was a serious practical problem in automobiles some twenty years ago, and it was solved by a series of constructional changes which were determined mostly empirically. Several papers and books at that time disclosed the theoretical reasons for these changes. Recently, the trouble has reappeared in aircraft landing wheels where the dimensions and constants are radically different from those in automobiles, thus reopening the problem.

7 Ship Propellers. The ship propeller, being a device with a small number of blades rotating in a field of nonuniform water velocity near the hull of the ship, is the cause of a variety of vibrations which are fairly well understood on the whole. However, many details still await solution, and many papers on such subjects as cavitation, the amount of entrained water, the cause of the self-excited condition known as "singing," have appeared in the recent past and are expected to continue so in the future.

8 Oil-Well Drill Shafts. Oil wells are now being drilled to depths of over 20,000 feet by rotating rods that are driven above ground and carry a drill tool from which the power is absorbed at the bottom of the hole. Such a shaft winds up many revolutions, buckles in a large number of sine waves, carries large stresses

Editor's Note: The first in a series of special articles, contributed to Applied Mechanics Reviews by international authorities, surveying important topics in applied mechanics. We are privileged to have Professor Den Hartog introduce this series with his authoritative appraisal of the field of dynamics and vibration.

due to its own weight, and is subjected to considerable friction from the sides of the hole. To the industry this presents difficulties that not only cause excessive costs, but even seem to impose a limitation on the depth that can be reached. To the theoretical investigator it offers a beautiful problem, which so far has been discussed primarily in the practical journals of the drilling industry. It is expected that references to this subject will appear in these REVIEWS more often than has been the case heretofore.

9 **Galloping Transmission Lines.** This phenomenon, when observed more than 20 years ago on ice-coated cables in a transverse wind, was explained as a stalled flow instability. However, no satisfactory method for preventing or damping the motion has ever been proposed, and electric transmission lines continue to be damaged by it every season. At present, research on actual and experimental lines is being conducted in several places, so that publications can be looked forward to.

10 **Torsional Vibrations of Engines.** This is an old problem on which hardly much new development is to be expected, although papers written on it continue to be numerous. Some twelve years ago an important new development was the centrifugal pendulum tuned absorber, which, for a while, overshadowed the older friction-type Lanchester damper. Recently, however, this Lanchester damper has acquired a new lease on life with the introduction of silicone fluids, of which the viscosity does not vary too much with the temperature. This makes possible a compact and durable construction which is now available on the market, ready made in a series of sizes. Several papers have appeared recently treating the response of silicone damped engines to excitations covering the entire speed range.

11 **von Kármán Vortices.** These curious eddies, which were first?) observed, described, and drawn by Leonardo da Vinci a few years before 1500, were discussed a few centuries later by Strouhal, Bénard and von Kármán, each of whom disclosed something more about them. It would seem, then, that the last word on the subject would have been spoken by this time. But during the last five years reputable authors have stated in print that the vortices do not occur for Reynolds numbers greater than 2,500 or 1,000,000 (a slight scattering of values). Also, large smokestacks recently were observed to sway violently in the wind at Reynolds numbers of over 3,000,000, so that more publications (even if not the last word) on this subject can be safely predicted.

Theoretical and Experimental Methods

(See also Revs. 79, 103, 148, 371)

1. **Milne, W. E., The remainder in linear methods of approximation,** *J. Res. nat. Bur. Stands.* **43**, 501–511, Nov. 1949.

One of the most important problems in numerical approximation is to determine the degree of approximation. Paper presents a systematic treatment of error for such diverse processes as interpolation, numerical differentiation, numerical integration, harmonic analysis, approximation by least squares, approximation by equating moments, and other applied operations. In all of these types of approximation, the first essential idea is to employ a basic set of linearly independent known functions $U_0(x)$, $U_1(x)$, ... $U_n(x)$, and to use an appropriately chosen linear combination of these functions as an approximation to given function. The coefficients in the approximating linear combination functions are now determined by some principle of equivalence. By suitable transformation, the remainder is put in a form in which it is possible, in many cases, to estimate its approximate magnitude. Several examples are carried out in paper and bounds are obtained for magnitude of error. Possibilities of securing unstable remainder terms by present methods, however, have not been fully explored. It is, for example, unknown whether these methods will give practically useful forms of remainder for case of polynomial approximation by least squares, for case of Fourier series, and more general Fourier type expansions.

Chi-Teh Wang, USA

2. **Greenwood, Robert E., Numerical integration for linear sums of exponential functions,** *Ann. Math. Statistics* **20**, 608–611, 1949.

A general method is presented for developing numerical integration formulas similar to Newton-Cotes formula but giving exact answers for the integral of functions other than polynomials. Method is used to determine coefficients in two types of formulas where integration of selected exponentials is exact. Estimates of error are given, as well as seven examples integrated by several methods. It appears that the symmetrical exponential method compares favorably with the Newton-Cotes method for some functions.

S. Levy, USA

3. **Berthod-Zaborowski, and Mineur, Henri, On the numerical calculation of double integrals** (in French), *C. R. Acad. Sci. Paris* **229**, 919–921, 1949.

Note deals with the numerical evaluation of a double integral by method of iterated integration and use of Gregory's or Gauss's formulas for evaluation of single integrals involved. Difficulties may arise due to form of region over which double integration is to be performed. Authors treat three cases, showing how difficulties can be circumvented.

Courtesy of Mathematical Reviews W. E. Milne, USA

4. **Bouzitat, Jean, On the numerical integration approached by the generalized Gauss method and on the extension of this method** (in French), *C. R. Acad. Sci. Paris* **229**, 1201–1203, 1949.

Author considers an extension of Gauss's generalized method of numerical integration to case where two of the abscissas are chosen beforehand (say as end points of interval of integration) and remaining points are determined so as to make the formulas exact for polynomials of as high degree as possible. He obtains formulas in case of weight function $x^\alpha(1-x)^\beta$ for interval (0, 1), weight $x^\alpha e^{-x}$ for interval $(0, \infty)$, and weight $e^{-x^2/2}$ for $(-\infty, \infty)$.

Courtesy of Mathematical Reviews W. E. Milne, USA

5. **Freudenthal, H., Mathematical problems of feed back** (in Dutch), *Nederl. Tijdschr. Natuurkunde* **15**, 275–281, 1949.

6. **Harrison, Joseph O., Jr., Piecewise polynomial approximation for large-scale digital calculators,** *Math. Tables and Other Aids to Computation* **3**, 400–407, 1949.

"Given the function $f(x)$ defined on the interval $[\alpha, \beta]$, a specified positive tolerance T, and a specified positive integer N," it is "required to divide $[\alpha, \beta]$ into subintervals $[c_{i-1}, c_i]$ ... and to determine nth degree polynomials $P_n{}^i(x)$ with $n \leq N$, such that the upper bound of $|f(x) - P_n{}^i(x)| \leqq T$ on $[c_{i-1}, c_i]$." Discussion is restricted to approximation by nth degree polynomials agreeing with $f(x)$ at $n+1$ points on $[c_{i-1}, c_i]$. Assuming $n+1$ points of agreement have the same relative positions on each subinterval, and these relative positions to be given, author develops a procedure for determining the subintervals so as to satisfy the required conditions. He shows that, under procedure given, the number of subintervals is a minimum when relative positions of points of agreement on each subinterval are the same as relative positions on interval $[-1, 1]$ of the zeros of the

Chebyshev polynomial "of the first kind" of order $n + 1$ defined by $T_{n+1}(u) = 2^{-1} \cos [(n + 1) \cos^{-1} u]$.
Courtesy of Mathematical Reviews T. N. E. Greville

7. Charp, S., Fourier coefficient harmonic analyzer, *Elec. Engrg.* **68,** 1057, 1949.

A harmonic analyzer which computes Fourier coefficients by evaluation of their defining integrals, using mechanical ball-and-disk integrators, and presenting results directly on counters. No change gears are required for computations up to 100th harmonic.
Courtesy of Mathematical Reviews S. H. Caldwell

8. Tanner, Louis, and Deming, W. Edwards, Some problems in the sampling of bulk materials, *Proc. Amer. Soc. Test. Mat.* **49,** 1181–1188, 1949.

Paper gives a partial answer to problem of sampling bulk materials for some quality characteristics. Sampling procedure considered in this paper was developed in connection with problems of evaluating wool quality. The procedures, suitably modified, might be of value in sampling of materials which are received in packages, bales, bags, etc. It still leaves open the question of how one can best sample such materials as shipments of coal or ore. A more detailed treatment of problems of this kind will be found in Deming ["Some theory of sampling," John Wiley and Sons, 1950, chapter 5]. Benjamin Epstein, USA

9. Souriau, J. M., Matrix and spinor calculus (in French), *O.N.E.R.A.* no. 42, 26 pp., 1950.

Expository article dealing with matrixes and spinors. All the results are available in prior well-known sources.
Y. L. Luke, USA

10. Dugas, René, History of mechanics (Histoire de la mécanique) (in French), Neuchatel, Switzerland, Éditions du Griffon, 1950. Swiss fr. 65.

11. Polya, G., How to solve it, Princeton, N. J.; Princeton University Press, 1948, xv + 224 pp. $3.

Book is a thorough specification of systematic steps and attitude of mind required to solve problems. It is concerned mostly with solving mathematical problems but gives a few examples of puzzles and everyday problems to show that the processes are of general effectiveness. Entire text is woven about a "list" of steps to be taken, the list being printed on both front and back covers. Discussion and examples are on an elementary (high school) level.

Author's purpose is laudable and he has prepared an ingenious and interesting book. It should be most valuable to teachers, particularly of scientific subjects, and to high school and college students. Readers with mature experience and some natural gift in solving problems will enjoy reading portions of the book but will probably conclude, with reviewer, that no set of rules or procedures can substitute for inherent natural ability or gift in analysis or synthesis. However, such readers will probably also agree with author that, should an individual have such inherent ability, it should be carefully developed at an early age and by a skilled teacher. Book should be of real value in such training.

Part III, by far the largest part of book, is entitled "Short Dictionary of Heuristic." It is far from short and, read as a whole, is excessively repetitious and discontinuous—just like reading any dictionary! Many of the separate sections, in effect, retreat much, or even all, the subject matter from different viewpoints and are of greatly uneven length. Moreover, author has not succeeded in choosing subtitles to make particular topics easily located, should reader really try to use this section as a dictionary or for reference.

Altogether, the reader—if he finishes this book—will lay it down with mixed feelings: high approval of the objective to teach method in solving problems, admiration for ingenious examples and clear exposition, some weariness from repetition, and resignation to the fact (not stressed by author) that problem-solving ability is a gift inherent in genes—not books.
K. W. Miller, USA

12. Owen, J. D., and Davies, R. M., High-speed recording by a rotating-mirror method, *Nature Lond.* **164,** p. 752, Oct. 1949.

Rotating-mirror principle of recording is employed to study a phenomenon of very short duration, namely, a torsional stress pulse in a steel cylinder. Light from a mercury flash tube is reflected from an optical flat polished on surface of cylinder and thence impinged on a mirror driven at approximately 1000 rps by an air turbine. Charles W. Gadd, USA

13. Mahl, Hans, Measurements of lengths and thicknesses in the electron microscope (in German), *Arch. Tech. Messen* **156,** T81–83, Aug. 1948.

14. Boothroyd, A. R., and Cherry, E. Colin., Analogue calculating machine for functions of a complex variable, *Nature* **163,** p. 687, 1949.

Rational functions of a complex variable are studied by analog methods. The analog used is a current-carrying conducting sheet on which are placed point sources of current to represent singularities. Exploring electrodes can be used to measure either the potential or the potential gradient at selected points or along various paths on the conducting sheet. These measurements furnish data which may be interpreted in terms of properties of the function represented.
Courtesy of Mathematical Reviews S. H. Caldwell, USA

15. Muskat, M., and McDowell, J. M., An electrical computer for solving phase equilibrium problems, *J. petrol. Tech.* **1,** 291–298, Nov. 1949.

Authors have designed a special purpose analog machine which solves phase equilibrium problems in the algebraic form $\Sigma a_i/(b_i + x) = 0$, $i = 1, 2, \ldots, n$. Machine is essentially a parallel sequence of resistance lines, each driven by an adjustable voltage source. One resistance member from each line composes a ganged set of variable resistances which represent variable "x." Right setting for ganged variable resistors and solution of problem are found when algebraic sum of currents flowing in lines is zero. Charts have been prepared for rapid conversion of equations of phase problem to forms desired for computer. Errors of not more than $^1/_2\%$ are claimed. Need for constant readjustment of driving voltages while problem is being put into machine points either to need of design improvements, such as use of isolating amplifiers, or to development of a technique of using machine which reduces these variations. Reviewer suspects use of servo devices, making solution automatic, would be desirable if machine could be extended to a greater range of application. Again, with ganged variable resistors, treated problem could also be solved on analog machines such as Reeves or Goodyear machines. M. G. Scherberg, USA

16. Laurent, Mariette, Table of the Dixon elliptic function for the interval 0–0. 1030 (in French), *Acad. Roy. Belgique Bull. Cl. Sci.* (5) **35,** 439–450, 1949.

In considering suitable projections for a map of the Belgian

Congo, the conformal representation of the sphere in an equilateral triangle appeared to be advantageous. Existing tables for this projection [O. S. Adams, "Elliptic functions applied to conformal world maps," *U. S. Coast and Geodetic Survey, Spec. Publ.* 112, Wash., D. C., 1925] were inadequate. The function involved is the Dixon elliptic function sm u. The main body of this paper is a detailed table giving sm u for $u = 0(.0010).1030$, 10 D, Δ_1, Δ_2, Δ_3, and the inverse table of u for sm $u = 0(.0010)$, 1030, 10 D, Δ_1, Δ_2, Δ_3. N. A. Hall, USA

17. Tables **of the function sin ϕ/ϕ and of its first eleven** derivatives, **by the Staff of the Computation Laboratory,** *The Annals of the Computation Laboratory of Harvard University,* **XXII**, Cambridge, Mass., Harvard University Press, 1949, xviii + 241 pp. $8.

As indicated in its title, this table gives values of the sine-integral ordinate $g(\phi) = \sin \phi/\phi$, and of its first eleven derivatives $(d^n/d\phi^n)g(\phi)$ for $(180/\pi)\phi = 0(\frac{1}{2})3599\frac{1}{2}$; in other words, argument is given in degrees. Values are to 9 decimals, usually about 8 significant figures, and accuracy to 0.7 of a unit in the final digit is claimed. The twelve function values for each ϕ are given across an opening, with 60 lines to a page, giving a good, useful, well-arranged, and compact table.

Courtesy of Mathematical Reviews J. C. P. Miller, England

18. Abramowitz, **Milton, Tables of integrals of Struve** functions, *J. Math. Physics* **29**, 49–51, 1950.

Values of $H_0(x)$, $H_1(x)$, $L_0(x)$, $L_1(x)$ are given to 6 decimals when $x \leq 5$, and 6 significant figures elsewhere for $x = 0(.1)10$, where the notation $y(x) = {}_0\int^x y(t)dt$ is used, and $H_\nu(x)$, $L_\nu(x)$ are the Struve and modified Struve functions, respectively.

Courtesy of Mathematical Reviews J. C. P. Miller, England

19. Grobner, **Wolfgang and Hofreiter, Nikolaus, Integral** tables. **First part: Indefinite integrals** (in German), Wien, Springer-Verlag, 1949, 166 pp.

Table is broken into three main parts: Rational integrands, algebraic irrational integrands, and transcendental integrands. Collected formulas are well arranged and complete enough for a wide variety of general usage, without book becoming cumbersome. Section on elliptic integrals is well done and includes integrals of all 3 kinds. Reviewer believes that mathematicians, physicists, and engineers who have need for mathematical tables will find this work a valuable contribution.

A. Devinatz, USA

20. **Horton, C. W., A short table of Struve functions and** of **some integrals involving Bessel and Struve functions,** *J. Math. Physics* **29**, 56–58, 1950.

Paper gives, for $x = 0(.1)10$, values of $H_n(z)$, $n = 2, 3, 4$, $C_n(z)$, $n = 1, 2, 3^*, 4^*$, $D_n(z)$, $n = 0, 1, 2, 3^*, 4^*$ to 4 decimals (3 decimals for $n > 6$ in cases indicated by an asterisk). In this $H_n(z)$ is a Struve function and $C_n(z) = {}_0\int^z z^n J_n(z)dz$, $D_n(z) = {}_0\int^z z^n H_n(z)dz$.

Courtesy of Mathematical Reviews J. C. P. Miller, England

21. **MacDonald, A. D., Tables of the function** $e^{-\alpha z/\gamma}M(\alpha; \gamma; z)$, *Research Laboratory of Electronics, Massachusetts Institute of Technology, Tech. Rep.* 130, ii + 10 pp., 1949.

This report gives 6-figure or 6-decimal values of the function $M(z) = e^{-\alpha z/\gamma} M(\alpha, \gamma; x)$ in which

$$M(\alpha, \gamma; x) = 1 + \frac{\alpha}{\gamma}\frac{x}{1!} + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{x^2}{2!} + \dots$$

is the confluent hypergeometric function; values are given for $\gamma = \frac{1}{2}(\frac{1}{2})\,2$, $\alpha = .001, .01, .05, .1(.1)1$, also .25 and .75 with $z = 0.1(.1)1(.5)8$. They were computed by the Joint Computing Group of the Massachusetts Institute of Technology.

Courtesy of Mathematical Reviews J. C. P. Miller, England

22. **Tables of the generalized exponential-integral functions, by the Staff of the Computation Laboratory,** *The Annals of the Computation Laboratory of Harvard University,* **XXI**, Cambridge, Mass., Harvard University Press, 1949, xxvi + 416 pp. $8.

This volume is a companion to two earlier volumes in the same series [Tables of generalized sine- and cosine-integral functions, parts I and II, Cambridge, Mass., Harvard University Press]. It gives values of the three functions:

$$E(a, x) = {}_0\int^x \frac{1 - \exp\{-(a^2+x^2)^{1/2}\}}{(a^2+x^2)^{1/2}}\,dx$$

$$Es(a, x) = {}_0\int^x \frac{\exp\{-(a^2+x^2)^{1/2}\}\cdot\sin(a^2+x^2)^{1/2}}{(a^2+x^2)^{1/2}}\,dx$$

$$Ec(a, x) = {}_0\int^x \frac{1 - \exp\{-(a^2+x^2)^{1/2}\}\cdot\cos(a^2+x^2)^{1/2}}{(a^2+x^2)^{1/2}}\,dx.$$

Six decimals are given, together with first differences both x-wise and a-wise. Argument ranges for both x and a are as follows, for the various tables: I. 0(.001).049; II. 0(.002).098; III. 0(.005)-.245; IV. 0(.01).99; V. 0(.02)1.98; VI. 0(.05)2.45; VII. 0(.1)4.9; VIII. 0(.2)9.8. The arguments x and a vary independently through ranges indicated. Δx and Δa suffice for interpolation in tables I and II, but second differences are needed for full accuracy in the other tables. The arrangement is better than for the earlier volumes mentioned above and is satisfactory for both a-wise and x-wise interpolation. The introduction outlines properties of the functions, methods of computation and of interpolation, etc. Some relations seem worth quoting: Since the Bessel function $K_0(a)$ is given by

$$K_0(a) = {}_0\int^\infty \frac{\exp\{-a(1+t^2)^{1/2}\}}{(1+t^2)^{1/2}}\,dt,$$

the function

$$\overline{E}(a, x) = \sinh^{-1}\frac{x}{a} - E(a, x) = {}_0\int^x \frac{\exp\{-(a^2+x^2)^{1/2}\}\,dx}{(a^2+x^2)^{1/2}}$$

is an incomplete Bessel function. Note also that

$$E(0, x) + \mathrm{Ei}(-x) = \ln x + \gamma$$

is the special case a.

Courtesy of Mathematical Reviews J. C. P. Miller, England

23. **Thorne, C. J., A table of harmonic and biharmonic polynomials and their derivatives,** *Utah Eng. Exp. Station,* Univ. Utah, 157 pp., 1950.

A polynomial is symmetric if even powers of the variable occur. A biharmonic polynomial satisfies the biharmonic equation $\nabla^4 w = 0, \dots$ [1]. Author has shown that in certain types of boundary-value problems (see Rev 1, 610), solutions to [1] can be taken in the form $z^n/n!, \dots$ [2], and $\bar{z}\, z^{n-1}/n!, \dots$ [3], where $z = x + iy$ and $\bar{z}$ is its complex conjugate. Note that [2] also solves the Laplacian. To facilitate computations involving these polynomials, values of [2] and [3] and their first three derivatives are tabulated to 10S along certain lines in a two-

dimensional unit square. The interval of x and y is .05. The range of n is 0 (1) 25. Y. L. Luke, USA

24. Ghizzetti, Aldo, Flow in a not homogeneous and anisotropic medium, *Ann. Soc. Polon. Math.* **22,** 1949, 195–200 1950.

Paper concerns uniqueness of solution of the general linear elliptic partial differential equation of second order in two variables and with constant coefficients, when boundary conditions to be satisfied are that the normal derivative of the solution vanishes on the two infinite edges of a semi-infinite strip, the function itself assumes prescribed values upon the finite edge, and the function vanishes in a suitable fashion at ∞. Author concludes that there is at most one such solution of the equation if and only if certain inequalities are satisfied by the five constants occurring in the problem. Treated problem was proposed to author by Litwiniszyn [cf. the same source, 185–194, 1950].

C. Truesdell, USA

25. Kucher, D. L., On some criteria for the boundedness of the solution of a system of differential equations (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **69,** 603–606, 1949.

Author considers problem of determining the necessary and sufficient conditions that must be imposed upon solutions of the vector-matrix equation $dy/dt = A(t)y$ in order that all solutions of $dz/dt = A(t)z + f(t)$ be bounded for vector functions $f(t)$ belonging to different function spaces such as L, L_p, and M. The first results of this type were obtained by Perron [*Math. Z.* **32,** 465–473, 1930] for the space M, using matrix transformation methods. A direct method was employed by Malkin [*Sbornik Nauchnikh Trudov Kazanskogo Aviatzionnogo Instituta* no. **7,** 1937], who showed that $||y(t)|| \leqq c_1 \exp(-c_2(t-t_0))\,||y(t_0)||$, $t \geqq t_0$, for all $t_0 \geqq 0$, c_1, c_2, fixed constants, was necessary and sufficient for $f \varepsilon M$. Banach space methods were introduced by Bellman [*Ann. of Math.* (2) **49,** 515–522, 1948] to extend Perron's result to L_p, and to difference and allied functional equations. Kucher reverts to Malkin's method to show that criteria of Bellman may be put into much simpler form, akin to that of Malkin's above, from which more information can be obtained concerning the solutions of $dy/dt = A(t)y$. R. Bellman, USA

26. Levinson, N., Bogert, B., and Redheffer, R. M., Separation of Laplace's equation, *Quart. appl. Math.* **7,** 241–262, 1949.

Type of problem with which paper is concerned is to determine all coordinate systems in which a given equation can be solved by separation of variables. Emphasis is on Laplace's equation. Discussion is confined to three dimensions. Following results are obtained: (1) For the Laplace equation $\Delta\Theta = 0$ in curvilinear coordinates (u, v, w) in Euclidean space to be directly separable into two equations, one for S and one for Z, when solution is $\Theta = R(u, v, w)S(u, v)Z(w)$ with fixed R, it is necessary and sufficient that surfaces w = constant are (a) orthogonal to surfaces u = constant, v = constant and (b) are parallel planes with a common axis, concentric spheres, spheres tangent at a common point, or one of the two sets of spheres generated by the coordinate lines when bicircular coordinates are rotated about line joining the poles or about its perpendicular bisector. (2) In first three cases $R = 1$ when, and only when, surfaces w = constant are parallel planes, planes with a common axis, or concentric spheres. (3) In these three cases, and these only, the wave equation separates in sense RSZ and hence, for the wave equation, $R = 1$ automatically. (4) For further separation of the equation found above for S, when $S = X(u)Y(v)$, so that solution is now $RXYZ$, it is necessary and sufficient that the coordinates be toroidal, or such that the wave equation so separates, or any inversions of these. (5) Coordinates where wave equation so separates, that is, admits solutions $RX(u)Y(v)Z(w)$, are only the known cases where this happens with $R = 1$, namely, degenerate ellipsoidal or paraboloidal coordinates. (6) In these cases, but only these, $R = 1$ for the Laplace equation too. (7) Coordinates for RSZ or $RXYZ$ separation of Laplace equation have the group property under inversion. (8) In all cases R can be found by inspection of the linear element.

Courtesy of Mathematical Reviews M. Pinl, USA

27. El'šin, M. I., Qualitative problems on the linear differential equation of the second order (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **68,** 221–224, 1949.

Author discusses problem of finding a representation of the solutions of $x'' + p(t)x' + q(t)x = 0$ in the form $x = a(t)\cos(b(t) + \epsilon)$ by the use of suitable transformations of variable. He then points out the application of this result in determining oscillatory properties of solutions. R. Bellman, USA

28. El'šin, M. I., On the decremental estimate of amplitudes (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **69,** 7–10, 1949.

Author has previously shown that under certain conditions, the general solution of $u'' + p(t)u' + q(t)u = 0$ can be put in the form $u = c_1 \exp\left(-\frac{1}{2}\,{}_{t_0}\!\int^t p\,ds\right) \cos\left[{}_{t_0}\!\int^s w\,ds + c_2\right]/|w|^{1/2}$, where w is determined by another functional equation of differential type. He now investigates the boundedness or unboundedness of the amplitude, $\exp\left(-\frac{1}{2}\,{}_{t_0}\!\int^t p\,ds\right)/|w|^{1/2}$. R. Bellman, USA

29. Gunther, N., On Stieltjes integrals and their application in mathematical physics (Sur les integrales de Stieltjes et leurs applications aux problèmes de la physique mathematique) (in French), New York, N. Y., Chelsea Publishing Company, 1949, iii + 494 pp. $4.95.

The original was published in *Trav. Inst. Phys.-Math. Stekloff* **1,** 1–494, 1932.

30. Friedman, Bernard, Note on approximating complex zeros of a polynomial, *Comm. Pure Appl. Math.* **2,** 195–208, 1949.

Author improves Lin's method for finding a linear or quadratic factor of a polynomial $f(x)$. Let $g(x)$ be the trial factor of $f(x)$: $f(x) = g(x)\cdot q(x) + r(x)$. Then an improved factor $g_1(x)$ is found by dividing $f(x)$ by $q(x)$ in ascending powers of x: $f(x) = q(x)\cdot g_1(x) + r_1(x)$. Convergence is better than in Lin's method. Author adds a generalization of synthetic division when divisor is a quadratic function.

Courtesy of Mathematical Reviews E. Bodewig, Holland

31. Massera, J. L., On Liapounoff's conditions of stability, *Ann. of Math.* (2) **50,** 705–721, 1949.

A stable solution $x(t) = (x_1(t), \ldots, x_n(t))$, say $x = 0$ for simplicity, of a system $\dot{x}(t) = X(x, t)$ of ordinary differential equations is called by Liapounoff asymptotically stable if there exists a $\delta > 0$ such that every solution whose initial conditions at $t = 0$ differ from 0 by less than δ tends to 0 as t tends to infinity. Present author introduces the stronger notion of equiasymptotic stability by requiring the existence of a positive δ such that the $\lim x(t) = 0$ as $t \to \infty$ be uniform for all initial conditions $|x(0)| \leq \delta$. The relationship between these two notions of asymptotic stability is investigated. In general they are different, but coincide for systems which are either of order 1, or linear, periodic, or autonomous. Several known theorems on asymptotic stability are strengthened, both by weakening the assumptions and by

showing that equiasymptotic stability is implied. These results lead to necessary and sufficient conditions for asymptotic stability when the system is either linear or periodic.

Courtesy of Mathematical Reviews F. Bohnenblust, USA

32. **Levinson, Norman, An ordinary differential equation with an interval of stability, a separation point, and an interval of instability,** *J. Math. Physics* **28,** 215–222, 1950.

The special differential equation $\epsilon d^2u/dx^2 + f(u, du/dx) = 0$ is considered, where $f = u - [4u'/(3+u'^4)]$. For the fixed initial conditions $x = 0$, $u = 8/19$, $u' = 2$, behavior of the solution $u = u(x, \epsilon)$ is compared with that of the solution $z = z(x)$ of the limiting differential equation $f(z, z') = 0$ which satisfies same initial conditions. The existence of a value $x_1 > 0$ is shown such that the solution $z(x)$ exists in $0 \leq x \leq x_1$ but cannot be prolonged beyond this value. On the other hand, the solution $u(x, \epsilon)$ tends to $z(x)$, as ϵ tends to $+0$, in the interval $0 \leq x < x_1$, but for $x > x_1$ it oscillates more and more rapidly, as ϵ tends to $+0$, with an amplitude approximately equal to one. In particular, it cannot tend to a limit as ϵ tends to zero. In the study of viscous incompressible flows around an obstacle, experiments have suggested the occurrence of regions of stability and of instability. Although example considered in this paper has no direct bearing on problem of viscous flows, its interest lies in the fact that it exhibits a simple ordinary differential equation in which such intervals of stability and of instability occur.

Courtesy of Mathematical Reviews F. Bohnenblust, USA

33. **Ivanov, A. V., Generalization of the formula for the operational representation of the product of two functions** (in Russian), *Prikl. Mat. Mekh.* **13,** 6, 663–664, Nov.–Dec. 1949.

Following theorem is proved: If $f(p) = L\{F(t)\}$, and $g(w, p) = L\{e^{wa(t)}G(t)\}$, where $L\{F\}$ is the Laplace transform of F, then $L\{F[a(t)] \cdot G(t)\} = (2\pi i)^{-1} {}_{\sigma-i}\int^{\sigma+i} f(w)g(w, p)dw$ with the real number σ suitably chosen. Author states that the particular case of this theorem with $a(t) = t$ was given by G. A. Grünberg [*C. R. (Doklady) Acad. Sci. SSSR (N.S.)* **40,** 141–143, 1943]. This particular case of the theorem, however, was given earlier by Bourgin and Duffin [*Amer. J. Math.* **59,** 489–505, 1937].

Courtesy of Mathematical Reviews H. P. Thielman, USA

34. **Labin, E., Operational calculus (Calcul operationel)** (in French), Paris, Masson & Co., 1949, 145 pp. **780 fr.**

35. **Hölder, Ernst, On the variational principle in mechanics of continua** (in German), *Ber. Verh. Sächs. Akad. Wiss. Leipzig. Math.-Nat. Kl.* **97,** no. 2, 13 pp., 1950.

Motion of a continuous medium is described by the three functions $b^i = b^i(x^\beta, t)$, where x^β are Eulerian coordinates referred to axes rotating with angular velocity $\omega_{\alpha\beta} = -\omega_{\beta\alpha}$, t the time, and b^i Lagrangian coordinates. In terms of these functions, density is $\rho = \rho_0 \det (p_\beta{}^i)$, with $\rho_0 = \rho_0(b^i)$ and $p_\beta{}^i = \partial b^i/\partial x^\beta$, and velocity is $u^\gamma = -a_i{}^\gamma p_0{}^i$ where $a_i{}^\alpha p_\beta{}^i = \delta_\beta{}^\alpha$ and $p_0{}^i = \partial b^i/\partial t$ $(x^0 = t)$. Medium is assumed to possess internal energy per unit volume $\Omega(p_\beta{}^i, b^i, s)$ where s is entropy-density, a function of b^i (adiabatic hypothesis). The central hypothesis is a Hamiltonian principle (a generalization of one due to Clebsch) which reads $\delta_j F(t, x^\beta; b^i, p_\beta{}^i)dxdt = 0$, where $F = \frac{1}{2}\rho u_\gamma u^\gamma + \rho\omega_{\delta\gamma}x^\delta u^\gamma - \Omega - \rho U$, U being potential energy of body forces per unit mass. With $\mu, \nu = 0, 1, 2, 3$, and other suffixes 1, 2, 3, the author defines an energy-momentum tensor $U_\mu{}^\nu = F\delta_\mu{}^\nu - p_\mu{}^i\pi_i{}^\nu$, where $\pi_i{}^\nu = \partial F/\partial p_\nu{}^i$, and establishes the identity $dU_\mu{}^\nu/dx^\nu - F_{x^\mu} = p_\mu{}^i(F_{b^i} - d\pi_i{}^\nu/dx^\nu)$. Then the Euler-Lagrange equations of the variational principle imply the four conservation laws $dU_\mu{}^\nu/dx^\nu - F_{x^\mu} = 0$. The equation of continuity is satisfied and the conservation laws for $\mu = 1, 2, 3$ give the equations of motion

$$\rho(du_\alpha/dt+2\omega_{\beta\alpha}u^\beta) = (d/dx^\beta)(\Omega\delta_\alpha{}^\beta-p_\alpha{}^i\partial\Omega/\partial p_\beta{}^i) - \rho\partial U/Lx^\alpha.$$

Specializations are made to a gas, with $\Omega = f(\rho, s, b^i)$, and in particular to its steady streaming. J. L. Synge, Ireland

36. **Stussi, Fritz, Numerical solution of boundary value problems by means of the equation of the funicular polygon** (in German), *Z. angew. Math. Mech.* **1,** 53–70, Jan. 1950.

Author develops method of solving second-order total differential equations with constant coefficients making use of string polygon for a loaded beam. Possibility is mentioned of extending method to certain equations of fourth order.

Harry D. Huskey, USA

37. **Narodetzkiĭ, M. Z., and Sherman, D. I., On a problem of conformal mapping** (in Russian), *Prikl. Mat. Mekh.* **14,** 209–214, 1950.

Let S be the doubly connected region bounded on inside by a circle of radius R, and on outside by a concentric ellipse with semi-axes a and b, $a > b$. Function (φz) which maps S conformally onto region $\rho_1 > |w| > \rho^a$ is obtained in form of an infinite series. A small number of terms of the series may be used to give a good approximation to $\varphi(z)$ when $c = R/b$ is a small number. Detailed computations are given for $c = .93$, $k = a/b = 1.25$, using 5 terms of the series, and for $c = .71$, $k = 3$, using 7 terms of the series. Relative error on boundary of ellipse is on the order of .003. It is stated that same method used to determine $\varphi(z)$ can be applied to determine the mapping function for a number of other elementary doubly connected regions.

A. W. Goodman, USA

38. **Bereis, Rudolph, Mechanisms to effectuate Joukowsky mapping** (in German), *Arch. Math.* **2,** 126–134, 1950.

Joukowsky employed conformal transformations to obtain the streamlines and equipotential lines in aerodynamic flow. One of these is given by $z' = z+a^2/z$. Author mechanizes this transformation by a double Peaucellier cell of twelve links. Ends of the cell slide along the real and imaginary axes. By cutting the linkage in half, number of links is halved to six without loss of operability. Several other variants of this mechanism are exhibited. In one of them, derivable from Hart inversor, parameter a is adjustable. M. Goldberg, USA

39. **Floyd, G. F., Jr., The use of autocorrelation functions in fire-control system design,** *Symp. appl. Autocorr. Anal. Phys. Problems,* 45–46, June 1949.

When a fire-control system is firing at a moving target, the hit-probability density depends on two types of errors, one predictable (dynamic following error and computer calibration), the other random (dispersion of bullets, random misalignments, noise—chiefly on the radar). Experiment shows the random error to be essentially normal. Mathematical analysis shows that its mean-square value depends only on the system function and the autocorrelation of the random system input.

Albert A. Bennett, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 58, 83)

40. Federhofer, K., On the graphical kinetostatic of plane gears (in German), *Österreich. Ing.-Arch.* **4,** 130–135, 1950.

This is an extension of the work of author and others on the graphical solution of plane kinetostatic problems. Displacement diagram is used to determine forces by a balance of moments on Joukowsky lever [see S. Timoshenko and D. H. Young, "Theory of Structures," London-NewYork, McGraw-Hill, 1945, p. 42] as well as velocity and acceleration diagrams. As an example of method, it is applied to determination of forces on cam shaft of an internal-combustion engine. M. Goldberg, USA

41. Berker, Ratip, Lectures on mechanics (Mekanik Dersleri), Vol. 1. Vectors. Kinematics, 2d ed., *Istanbul Technical University Publ.* no. 96, 1949, vii + 273 pp.

This book, the first volume of a general introductory treatment of mechanics, contains two parts. First part is concerned with vectors, and second part with kinematics. Treatment is thorough and detailed, although some rather important topics are omitted or relegated to exercises. Thus, in chapter on vector algebra, the triple scalar product $a\cdot(b\times c)$ is discussed in greatest detail, while the triple vector product $a\times(b\times c)$ is omitted from text and is only included among exercises. In part on kinematics, there is an unusually detailed discussion of cycloids, epicycloids, hypocycloids, evolutes, involutes, and other curves of this nature. D. C. Lewis, USA

42. Bottema, O., On the kinematic representation of Beth and its application to the central connecting-rod motion (in Dutch), *Euclides, Groningen* **25,** 253–256, 1950.

43. Hain, K., and Meyer zur Capellen, W., Kinematics (in German), Naturforschung und Medizin in Deutschland 1939–1946, Band 7, pp. 1–41. Dieterich'sche Verlagsbuchhandlung, Wiesbaden, 1948.

This survey summarizes published work in kinematics in indicated period, giving titles, authors, and complete references. Principal sources are *Maschinenbau, Z. Instrumentenkunde, Z. Verein Deutsch. Ingenieure, Z. angew. Math. Mech.*, and *Automobiltech. Z.* The tradition established by F. Reuleaux is continued in books by R. Franke, R. von Voss and K. Rauh in their systematic synthetic classification of mechanisms. Others, including authors of this survey, have derived the relation between number of degrees of freedom and numbers of members and joints, paths of special points like instantaneous centers and centers of gravity as well as paths of arbitrary points, and graphical and mechanical methods used by engineers for determination of velocities and accelerations. Beyer published several papers on extension of plane results to space mechanisms. Equivalence of various pairs of mechanisms are established. Mechanisms are constructed to take assigned orientations and positions. Several new books on toothed gears have appeared as well as papers on gearing, variable speed drives, cam drives, and intermittent drives. M. Goldberg, USA

44. Hain, Kurt, Four-bar linkage synthesis for prescribed orientation of locus tangents (in German), *Ingen.-Arch.* **18,** 3, 141–150, 1950.

In the title problem, of interest in the design of mechanisms for tracing given curves, x positions of a locus point of the floating link and y orientations of the tangents to its path at one or more of these positions are prescribed. Whenever $x + y \leq 7$, solutions are possible, and several are demonstrated in detail. Extensive use is made of "point position reduction," i.e., one of the linkage points is chosen in a particular position, relative to given positions of the locus point, such that either the geometrical solution is greatly simplified or the permissible number of prescribed conditions is increased. It is claimed that the method may be extended to multilink (above four) mechanisms. G. H. Nothmann, USA

45. Carter, B. C., Analytical treatment of linked levers and allied mechanisms, *J. roy. aero. Soc.* **54,** 247–252, 1950.

Two cranks, which rotate about nonintersecting axes, are joined by a ball-ended connecting rod. The relative rotations and velocities of the cranks are derived. A similar derivation is obtained for shafts joined by Hooke's universal joints. M. Goldberg, USA

46. Hain, Kurt, The development of switch gears (in German), *Z. Ver. dtsch. Ing.* **91,** 589–596, Nov. 1949.

After a brief survey of types of mechanisms which convert continuous rotation into intermittent rotation, author discusses "gear bar mechanisms" (bar linkages with gears at two or more of the joints; one gear is integral with one of the bars; one of the bars serves as driver, one of the gears as follower). He solves following problems: (1) Given a bar linkage, to select proper gear ratios to produce dwell of follower at prescribed positions and near-dwell over intermediate portions of cycle. (2) Given a bar linkage, to select proper gear ratios to produce near-dwell of follower over a prescribed portion of the cycle. (3) Synthesis of gear bar mechanisms to produce near-dwell of follower over prescribed portions of the cycle. Methods given are simple and direct and should be helpful in rational design of mechanisms for obtaining an intermittent output from a continuous input motion. G. A. Nothmann, USA

47. Manacorda, Tristano, On a non-linear differential equation of dynamics of a point (in Italian), *Ist. Lombardo Sci. Lett. Rend. Cl. Sci. Mat. Nat.* (3) **11(80), 1947,** 85–98, 1949.

Behavior of solutions of nonlinear equation $\ddot{y} + 2\alpha\dot{y} + f(y, t)y = 0$ is studied by classical methods of theory of Sturm. It is assumed that $|f|$ is bounded, which implies the existence of the solutions for all values of t. Two main results are: If there exists an ω such that $f(y, t) \leqq \omega^2 < \alpha^2$ then every solution y can vanish at most once; if there exists ω such that $0 \leqq \alpha^2 < \omega^2 \leqq f(y, t)$ then every solution has infinitely many zeros. Author restricts himself unnecessarily to a more special form of $f(y, t)$. It is erroneously stated on page 89 under 4a that the solutions y do not vanish for $t > 0$. [In notation of author, case $\beta = 0, \gamma =$ constant, $\gamma < \alpha^2/4$ leads to a counter example. Differential equation becomes linear equation $\ddot{y} + \alpha\dot{y} + \gamma y = 0$ whose solutions may well vanish for $t > 0$.] As stated above it is only true, under conditions assumed, that they vanish at most once. As a result of this error some of later proofs must be changed slightly.

Courtesy of Mathematical Reviews F. Bohnenblust, USA

48. MacColl, L. A., A theorem concerning the positions of equilibrium of a rigid body, *Quart. appl. Math.* **7,** 472–473, 1950.

Let B be a rigid body, of any form, which is free to rotate about a fixed point P. Let B be subjected to forces which are derived from a potential function V, which is an arbitrary single-valued function, of class C^2, of the coordinates of B. Then B has at least four distinct positions of equilibrium. Theorem is an immediate consequence of following known theorems. (1) Configuration space of rigid body is homeomorphic with real three-dimensional projective space; (2) number of distinct stationary points of a real single-valued function, of class C^2, defined over a closed manifold M is at least equal to category of M.

(3) category of real n-dimensional projective space is $n+1$.

Courtesy of Mathematical Reviews O. Bottema, Holland

49. Kasner, **Edward, and De Cicco, John, Physical families in conservative fields of force,** *Proc. nat. Acad. Sci. Wash.* **35,** 419–422, 1949.

Authors continue their discussion of what they call physical families of curves. [For a definition see the same source **34,** 68–72, 1948.] They now consider case of motion on a plane or other surface in a conservative field of force, stating various simple basic properties of the families of curves.

Courtesy of Mathematical Reviews L. A. MacColl, USA

50. Sonntag, **G., Calculations of stresses and slipping for rolling of deformable bodies** (in German), *Z. angew. Math. Mech.* 30, 73–83, 1950.

Calculations of stresses, slipping, and friction for rolling spheres are made. They neglect change of shape of cross section, but are shown to be good approximations by use of rigorously obtained upper and lower bounds.

Courtesy of Mathematical Reviews P. Franklin, USA

51. Bottema, **O., On the small vibrations of nonholonomic systems,** *Proc. Kon. Ned. Akad. Wet.* **52,** 848–850, Oct. 1949.

Author denies statement by Whittaker ("Analytical Dynamics," pp. 221–222) that for small vibrations, there is no essential difference between systems with holonomic and nonholonomic constraints. He considers a system in a potential field which does not vanish at point of equilibrium, i.e., one with a linear potential function rather than the quadratic potential function assumed by Whittaker, and shows that Whittaker's argument on basis of smallness of Lagrange multipliers must be modified for this case. Since this modification introduces into the linearized equations of motion the coefficients c_{ij} of the linear terms in the coefficients of nonholonomic constraint, and since it is not necessarily true that the c_{ij} are symmetric, the modification is indeed a departure from the holonomic case. Author does not observe, however, that a system cannot have an equilibrium point at which potential gradient fails to vanish unless constraint is such as to permit local motion only in directions orthogonal to potential gradient. Thus, only a certain class of nonholonomic constraints satisfy conditions under which the argument of author proceeds, and remaining cases of nonholonomic constraint are correctly treated as described by Whittaker.

Robert E. Roberson, USA

52. Aizerman, **M. A., On a problem concerning the stability "in the large" of dynamical systems** (in Russian), *Uspehi Matem. Nauk* (*N.S.*) **4,** 4(32), 187–188, 1949.

Consider the linear system: $dx_1/dt = \sum_{j=1}^{n} a_{1j}x_j + ax_k$, $dx_i/dt = \sum_{j=1}^{n} a_{ij}x_j$, $i = 2, 3, \ldots n$, where it is assumed that characteristic roots of coefficient matrix have uniformly negative real parts for $\alpha < a < \beta$. Then if $f(x)$ is a continuous function satisfying the condition $\alpha x \leqq f(x) \leqq \beta x$, $f(0) = 0$, the solution $x_1 = x_2 = \ldots = x_n = 0$ of the system $dx_1/dt = \sum_{j=1}^{n} a_{1j}x_j + f(x_k)$, $dx_i/dt = \sum_{j=1}^{n} a_{ij}x_j$, $i = 2, 3, \ldots, n$ is stable. Result can be generalized. R. Bellman, USA

53. Poppinga, **R., Efficiency of planetary transmissions** (in German), *Ingen.-Arch.* **18,** 1, 39–52, 1950.

Seventeen planetary transmissions, of forms commonly used, are analyzed for dependence of efficiency on transmission ratio. Only tooth friction is included, windage and bearing losses being omitted. It is assumed that, in the ordinary gear train obtained by inversion of the planetary transmission, the efficiencies will be equal at all tooth meshes. For some forms, efficiency of planetary transmission is shown to decrease rapidly with increasing transmission ratio, while, for others, efficiency is nearly independent of transmission ratio. No experimental results are shown.

A. S. Hall, USA

54. **Arzhaníkh, I. S., Nonholonomic dynamical systems possessing a kinetic potential** (in Russian), *Dokladí Akad. Nauk SSSR* (*N.S.*) **65,** 809–811, 1949.

Necessary and sufficient conditions under which equations of a nonholonomic system possessing a potential can be presented in Lagrangian form are given, using tensor analysis, as integrability conditions of a system whose coefficients are expressed in terms of the explicit equations of motion. There are some misprints.

Courtesy of Mathematical Reviews G. Y. Rainich, USA

55. **Walker, P. B., Measurement of mechanical actions by electrical-resistance methods,** *Aer. Res. Counc. Rep. Mem.*, R. & M. 2388, June 1946, 8 pp., 1950.

In general discussion of methods of employing electrical resistance elements in measurement of various force applications, consideration is given to elements on which entire load is applied, in contrast to bonded resistance-wire strain gages in common use which assume only negligible fraction of impressed force. Design suggestions are presented for such elements. Effects of fluid pressure, as well as tensile and other types of deforming actions, on resistance elements are outlined briefly. Following a discussion of elementary elasticity theory in relation to electrical resistance, various unit strain factors are given in tabular form as a function of Poisson's ratio and type of loading.

P. D. Wickersham, USA

56. **Schnarbach, Kurt, Significance and application of the transmission angle** (in German), *Z. Ver. dtsch. Ing.* **91,** 603–606, Nov. 1949.

Author first recalls importance of the transmission angle (complement of pressure angle) in problems of cam mechanisms. From a known graphical method, he deduces a chart giving, for different motion laws, minimum transmission angle in function of: total follower motion, corresponding rotation angle, and mean radius of cam. Use of chart is extended to cams with swinging arms and also to cylindrical cams.

D. De Meulemeester, Belgium

57. **Favre, Henry, A course in mechanics. III: Selected topics (Cours de mecanique. III. Chapitres choisis)** (in French), Paris, Dunod, 1949, 476 pp. 3,160 fr.

For third volume of his "Course in mechanics," author has selected a series of lectures he gave in past years to graduate students at Zurich Polytechnic School. Principal object of the book which, from a mathematical point of view, should be considered rather elementary, is to give engineering students a general idea of principal methods currently used in modern applied mechanics and some of their most important practical applications.

Volume is divided into three parts: First is devoted to mathematical theory of elasticity: two-dimensional elasticity, work of deformation; principle of virtual displacements and some of the most important applications; bending of plates. Second part deals with vibrations of elastic bodies: vibrations of flexible strings, of bars, of membranes, and plates. Third part is devoted to hydrodynamics: dynamics of perfect fluids with application to wave propagation and to the water-hammer problem, and finally an introduction to dynamics of real fluids. Last

chapter of book treats dimensional analysis. This new contribution by Professor Favre in the field of applied mechanics will be welcomed by all interested in the subject and, particularly, by engineering teachers. E. Volterra, USA

Gyroscopics, Governors, Servos

(See also Rev. 362)

58. Neĭmark, Yu. I., *D*-decomposition of the space of quasipolynomials (on the stability of linearized distributive systems) (in Russian), *Prikl. Mat. Mekh.* **13,** 349–380, 1949.

A sum $Q(z) = \sum_{k,s=0}^{m,n} a_{ks} z^k e^{\tau_s z}$ with complex coefficients a_{ks} and real exponents τ_s, is called a quasipolynomial. Consider a family of quasipolynomials depending on some parameters; by a D-decomposition of this family is meant a decomposition of the parameter space into the sets $D(0) + D(1) + \ldots\ldots + D(\infty)$, where $D(l)$ consists of the parameter points for which $Q(z)$ has precisely l zeros with positive real part. Author is interested in the number of roots of $Q(z) = 0$ in the right half plane, a question of importance in the study of stability of certain dynamical systems. For this purpose he investigates the D-decomposition when parameter space consists of coefficients and exponents of Q, as well as when exponents are fixed and parameter space is the space of coefficients. A more detailed study is made of the D-decomposition of the complex w-plane and of the real (u, v)-plane when $Q(z) = A(z) + wB(z)$, and $Q(z) = uA(z) + vB(z) + C(z)$, respectively ($A$, B, and C being given quasipolynomials). Method of study centers on the consideration of how a continuously varying parameter point can pass from one $D(l)$ set to another.
Courtesy of Mathematical Reviews A. Dvoretzky, USA

59. Kats, A. M., On the approximate solution of nonlinear differential equations of the second order (in Russian), *Prikl. Mat. Mekh.* **14,** 111–113, 1950.

The equation dealt with is

$$\ddot{x} + h(\dot{x}, x)\dot{x} + k^2(\dot{x}, x)x = 0 \qquad [1]$$

which is of importance in many applications to mechanics and radiotechnique. It includes among others van der Pol's equation. In general only approximate solutions are possible. Main interest is in nearly periodic solutions

$$x = \alpha \sin ({}_0\int^t \omega dt + \epsilon) \qquad [2]$$

To make $\dot{x}$ have the same form as if α, ω, ϵ were constant, one imposes $\dot{\alpha} \sin ({}_0\int^t \omega dt + \epsilon) + \alpha\dot{\epsilon} \cos ({}_0\int^t \omega dt + \epsilon) = 0$. Assuming first ω constant and following van der Pol, α and ϵ slowly varying functions of t (i.e. $\dot{\alpha}$, $\dot{\epsilon}$ small), one obtains the approximate system integrable by quadratures:

$$\dot{\alpha} = -(\alpha/2\pi)\, {}_0\int^{2\pi} h(\alpha \sin u, \alpha\omega \cos u) \cos^2 u \, du \qquad [3]$$

$$\dot{\epsilon} = (1/2\pi\omega)\, {}_0\int^{2\pi} k^2(\alpha \sin u, \alpha\omega \cos u) \sin^2 u \, du - \omega/2 \qquad [4]$$

If ω constant and ϵ small are inadequate assumptions, one may assume ω variable but $\dot{\omega}$ small and the average of $\dot{\epsilon}$ over a period zero. One obtains first (4) and also (3) but with $\dot{\alpha}$ replaced by $\dot{\alpha} + \dfrac{\alpha\dot{\omega}}{2\omega}$. From this one derives the system

$$\omega^2 = (1/\pi)\, {}_0\int^2 k^2(\alpha \sin u, \alpha\omega \cos u) \sin^2 u \, du$$

$$\dot{\alpha}\left(1 + \frac{d}{2\omega}\frac{d\omega}{d\alpha}\right) = -(\alpha/2\pi)\, {}_0\int^{2\pi} h(\alpha \sin u, \alpha\omega \cos u) \cos^2 u \, du$$

which may also be integrated by quadrature. As an example, a certain regulator with nonlinear servomotor gives rise to the equation $2T\ddot{\sigma} + 3|\sigma|^{1/2}\cdot\dot{\sigma} + 2|\sigma|^{1/2}\sigma = 0$, with the approximate solution $\sigma = (24 \cdot 2T^2/(t + C)^2) \sin \{3(2(t + C))^{1/2} + \epsilon\}$, where C is an arbitrary constant. The system has then an oscillation with frequency and amplitude both $\rightarrow 0$ whatever C.
S. Lefschetz, USA

60. Kulebakin, V. S., On the behavior of continuously perturbed automatized linear systems (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **68,** 855–858, 1949.

Consider a real system

$$\Sigma a_{ij}(D)x_j = \begin{cases} f(t), & i = 1, \\ 0, & i > 1, \end{cases} \quad i, j = 1, \ldots, n \qquad [1]$$

$D = d/dt$, where coefficients are real quadratic in D. Let it be reduced to a similar canonical form

$$\Sigma b_{ij}(D)x_j = c_i(D)f(t) \qquad [2]$$

where the b_{ij} and c_i are new polynomials of degree not exceeding $\mu(n)$. It is supposed that x_1 is the variable to be regulated. Author considers certain continuous approximations $cf(t) = A(t) + \theta\epsilon$, $|\theta| < 1$, ϵ small, replaces [2] by an approximation $b_{ij}\xi_j = A(t)$ and discusses the possible error for various choices of A. He also notes that, when A is a polynomial, then ξ_1 satisfies a homogeneous equation $f(D)\xi_1 = 0$, which is particularly convenient. S. Lefschetz, USA

61. Popovskiĭ, A. M., On the freedom of choice of the parameters of autonomic processes of regulation of several reciprocally related quantities (in Russian), *Avtomatika i Telemekhanika* **10,** 401–423, 1949.

Author sets up equations governing linear control mechanisms which regulate several variables simultaneously.
Courtesy of Mathematical Reviews E. N. Gilbert, USA

62. Bulgakov, B. V., and Litvin-Sedoi, M. Z., On a problem of automatic regulation with nonlinear characteristic (in Russian), *Avtomatika i Telemekhanika* **10,** 329–341, 1949.

Let φ, θ be coordinates for basic system to be regulated and ζ a coordinate for regulator (servomotor). Let basic differential equations be

$$T^2\ddot{\varphi} + U\dot{\varphi} + k\theta + \zeta = 0; \quad S\dot{\theta} + \theta = S\dot{\varphi}$$

$$V^2\dot{\zeta} + W\zeta = H(\psi); \quad Y^2\ddot{\varphi} + X\dot{\varphi} + l\varphi - \zeta/m = \psi$$

Here $H(\psi)$, the characteristic of the servomotor, is a nonlinear function very close to a broken line made up of a vertical segment upwards from the origin followed by an infinite horizontal ray to the right. The other Latin letters are constants, all positive except X and k, and their physical significance is more or less indicated by the equations. Since $H(\psi)$ is small, author replaces it by another function written $\mu H(\psi)$, μ small. He then applies perturbation method and discusses self-oscillations of the system and their amplitudes and stability for μ small. Effect produced by varying the parameters of servomotor are also considered [see B. V. Bulgakov, *Prikl. Mat. Mekh.* **4,** 4, 1942, and 10, 3, 1946]. S. Lefschetz, USA

63. Dubois-Violette, Pierre-Louis, Study of the influence of time of propagation upon the stability of regulating servomechanisms by the method of fusion of roots (in French), *C. R. Acad. Sci. Paris* **230,** 1499–1501, 1950.

Remarks on the stabilizing of servomechanisms which involve a time lag.
Courtesy of Mathematical Reviews P. Franklin, USA

64. **Fel'dbaum, A. A., Integral criteria for the quality of a regulation** (in Russian), *Avtomatika i Telemekhanika* **9**, 3–19, 1948.

Consider the system $dx/dt = Ax$, where A is a constant matrix whose characteristic roots have negative real parts. Let $V = x'Bx$ be a Liapounoff form, i.e., $dV/dt < 0$. In connection with automatic regulators, the integral ${}_0\int^{\infty} V dt$ is of importance. Author discusses various examples derived from electrical systems.

R. Bellman, USA

65. **Hänny, Jost, On the calculation of regulating phenomena in linear systems (Über die berechnung der Reguliervorgange in linearen systemen)** (in German), Thesis, Eidgenössische Technische Hochschule, Zurich, 1947, ii + 254 pp.

Book is a clearly and competently written introduction to modern theory of linear servomechanisms and regulators. A general descriptive survey of subject is given in introduction. Chapter I is devoted to a discussion of stability of feedback systems, the discussion centering around Nyquist stability criterion. Arrangement and treatment of material testifies to author's sound understanding of fundamentals of the subject. Responses of servomechanisms to given input signals are treated in chapter II; and in chapter III author sets up fundamental equations of motion for a number of particular systems, mostly regulators to control speeds of turbines. A short fourth chapter gives a résumé of the Laplace transformation theory which is used in calculation of transient responses. Fundamentally, book is quite similar to several others which have appeared in the U. S. A. recently. It differs from most of the other books in that it places less emphasis upon general theory, and more upon properties of particular systems, such as regulators mentioned above. Author is very much interested in procedures of analytical and numerical computation, and he discusses these at great, perhaps too great, length.

Courtesy of Mathematical Reviews L. A. MacColl, USA

66. **Solodovnikov, V. V., The frequency-response method in the theory of regulation (a survey)** (in Russian), *Avtomatika i Telemekhanika* **8**, 65–88, 1947.

67. **Leonhard, A., Criterion of stability especially for regulation circuits at prescribed quality of stability** (in German), *Arch. Elektrotechnik* **39**, 100–107, 1948.

A method is described of testing whether the damping exceeds a given amount. It depends on transforming the roots of the characteristic equation.

Courtesy of Mathematical Reviews P. Franklin, USA

68. **Letov, A. M., Strictly unstable regulating systems** (in Russian), *Prikl. Mat. Mekh.* **14**, 183–192, 1950.

Author considers the stability of the system

$$\dot{\eta}_k = \sum_{\alpha=1}^{n} b_{k\alpha}\eta_\alpha + \eta_k\xi,\ \dot{\xi} = f(\sigma), \quad \sigma = \sum_{\alpha=1}^{n} p_\alpha\eta_\alpha - \xi$$

by the method of Lur'e [*Prikl. Mat. Mekh.* **9**, 353–367, 1943].

Courtesy of Mathematical Reviews J. G. Wendel, USA

Vibrations, Balancing

(See also Revs. 103, 500)

69. **Morduchow, Morris, A theoretical analysis of elastic vibrations of fixed-ended and hinged helicopter blades in hovering and vertical flight,** *Nat. adv. Comm. Aero. tech. Note* 1999, 61 pp., Jan. 1950.

Purpose of paper is to present, under one cover, in a brief and simple fashion, a comprehensive theoretical analysis of frequency and damping characteristics of elastic helicopter rotor blades in hovering and vertical flight, when they perform small vibrations about a state of equilibrium. Analysis and general formulas are developed for hinged and fixed-end blades having any taper and cross-sectional distribution, with the restrictions that shear center and cg coincide, and that flapping and lagging do not affect blade pitch.

Explicit methods and formulas for calculation of natural frequencies and logarithmic decrements of principal modes are developed, from which eleven general conclusions are "rigorously" drawn. Flutter of helicopter blades, which may occur when blades are unbalanced, is briefly considered on basis of quasi-stationary flow, and simple criteria are derived for stability of coupled torsional and flapping vibrations in such cases.

Several questions are considered here which appear not to have been thoroughly investigated before. These include: Effects of end conditions on centrifugal-force contributions to natural bending frequencies; effect of free torsional vibrations on flapping and lagging vibrations of mass-balanced blades; effects of Coriolis, centrifugal and aerodynamic coupling between flapping and lagging vibrations of a blade; and a comparison between aerodynamic and internal damping in the principal vibration modes of rotating blades in bending and in torsion.

Wm. C. Johnson, Jr., USA

70. **Fumi, F., and Carrassi, M., Integration of the differential equation of vibrating strings in vacuum by means of functional operators with one variable** (in Italian), *Atti Accad. Ligure* **5**, 42–67, 1949.

A detailed discussion by means of operational methods of the partial differential equation $u_{xx} - a^{-2}u_{tt} = 0$ of the vibrating string. Only difference from presentation of subject in text-books is that the formal application of Heaviside calculus is justified by the theory developed by Sbrana and first named author [same vol., 7–33, 1949; Sbrana, *ibid.*, 173–186, 187–200, 201–217, 1949; *Boll. Un. Mat. Ital.* (3) **4**, 34–40, 1949] rather than by Laplace transform methods.

Courtesy of Mathematical Reviews A. Erdélyi, USA

71. **Covezzoli, P., On forced vibrations of an elastic beam in hereditary regime** (in Italian), *Atti Sem. Mat. Fis. Univ. Modena* **3**, 261–264, 1949.

Author takes up problem of a beam vibrating in accord with V. Volterra's theory, as treated recently by E. Volterra [see Rev **3**, 1439]. Using a method indicated by Graffi [*Nuovo Cimento* (8) **5**, 53–71, 1928], she obtains a general formula for change of amplitude and phase in a single harmonic component in terms of the Fourier transform of the kernel. E. Volterra's result for case when kernel is a sum of exponentials is included as a special case.

C. Truesdell, USA

72. **Giangreco, Elio, Vibrations of plates with ribs. I, II** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **8**, 34–38; 113–115, 1950.

Author extends results of Tolotti and Grioli [*Giornale del Genio Civile*, no. 6 = Pubbl. Ist. Appl. Calcolo no. 258, 1948] and his own [forthcoming] concerning static deflection w of elastic plates reinforced with ribs to the elastodynamic case. He limits his attention, however, to problem of a freely vibrating rectangular

plate with two central ribs rigidly attached thereto which intersect one another orthogonally. Employing simplified method developed by Tolotti and Grioli [op. cit., chap. 2] for case of plates reinforced by rigidly attached ribs, in conjunction with the principle of D'Alembert, and confining himself to case when $w = w(x, y) \cos \omega t$, author derives homogeneous integrodifferential equation which governs w. By developing the kernel therein in terms of its characteristic functions and terminating the series after a finite number of terms, he obtains successive approximations to the natural frequencies of vibration of plate by standard procedures. A few preliminary results for fundamental frequency are included.

In II, author presents in detail results announced in I concerning the fundamental frequency by employing a known result of thin plate theory [Nádai, A., "Die elastischen Platten," Berlin, Springer, 1925, p. 174]. He uses his approximation procedure to compute the fundamental frequency of above-mentioned plate in first and second approximation. A numerical example is included to illustrate rapid convergence of method.

Courtesy of Mathematical Reviews A. W. Sáenz, USA

73. Itokawa, Hideo, A new method of solving the problem of the vibration of plates (in Japanese), *J. Soc. appl. Mech. Japan* **2,** 96–98, 110, 1949.

The general solution of the equation of vibration of plates is formed by a series of Bessel functions and circular functions in polar coordinates. Frequencies of a rectangular plate calculated by this method showed good results, and frequencies of a triangular plate were calculated anew. From author's summary

74. Pignedoli, Antonio, Vibration frequencies on an elastic membrane with fixed epicycloidal boundary (in Italian), *Ann. Mat. Pura Appl.* (4) **30,** 291–307, 1949.

Author presents a method for approximating the frequencies of vibration of an elastic membrane held fixed along its epicycloidal boundary (supposed free of loops and cusps). Eigenvalue problem consists in determining eigenvalues μ^2 of the partial differential equation $\partial^2 u(x, y)/\partial x^2 + \partial^2 u(x, y)/\partial y^2 + \mu^2 u(x, y) = 0$, subject to boundary condition $u = 0$ on boundary. Use is made of known conformal transformation of the complex number plane onto itself which maps interior of (exterior of) unit circle onto interior of (exterior of) the epicycloid. Investigation leads to introduction of certain transcendental functions, analogous to Bessel functions, which are termed epicycloidal functions.

Courtesy of Mathematical Reviews J. B. Diaz, USA

75. Henderson, J., and Eliffe, C. E., The development of a spring foundation for forge hammers, *N. E. Coast Instit. Eng. & Shipb.* **66,** part 3, 155–182, Jan. 1950.

To isolate vibrations, each forge hammer with its foundation block was suspended on four leaf springs. An analytical solution is given for determining the necessary weight of foundation block and the stiffness of the springs; this is based on idealizing the system as three masses connected by two springs. Also discussed are dynamic properties of rubber pad under anvil, propagation of stress waves through whole assembly, and forces set up in steel mounting block. Details of construction and installation for two hammers are described. Dana Young, USA

76. Haringx, J. A., On highly compressible helical springs and rubber rods, and their application for vibration-free mountings, V, *Philips Res. Rep.* **4,** 375–400, Oct. 1949.

This is a continuation of author's analysis of resilient mountings and vibration absorbers, with particular reference to instrument mountings. Consideration is first given to a so-called two-dimensional mounting without absorber in which a mass may vibrate in one vertical plane only. Equations are derived for the general case without damping, and simplification is made for case having elastic supports symmetrical about a vertical axis, effects due to gravity being also considered. Equations are then derived for two-dimensional mounting with viscous damping. General three-dimensional case (where vibration may take place in all planes) is treated for a mass without absorber, and it is shown that under certain conditions problem is reduced to a system of one- and two-dimensional cases. Analysis is next extended to case of a two-dimensional mounting with vibration absorber (auxiliary mass). Simplification of equations is made for case where there is a vertical axis of symmetry and where the so-called centers of elasticity of main mass and auxiliary mass coincide. Conclusions are drawn regarding design of such a mounting for forced vibrations at high frequencies and also from standpoint of damping free oscillations due to transient impulses.

A. M. Wahl, USA

77. Hochrainer, A., The elastic mounting of a rigid body (in German), *Österreich. Ing.-Arch.* **3,** 247–261, 1949.

Paper treats the essentially elementary problem of finding vibration modes of small free and forced hormonic oscillations of a spring-mounted rigid body. Tensor notation is employed. Two illustrative examples are given. The problem is a special case of one treated by Lagrange [*Mécanique Analytique,* part 2, section 5, 1st ed., 1788; part 2, section 6, 4th ed., 1888]. Reviewer is not convinced by author's statement that by means of this procedure the modes of the system "can be calculated in a much clearer form than with methods used up to the present time."

Courtesy of Mathematical Reviews E. Pinney, USA

78. Mansa, J. L., Damped vibrations of a two-mass system, *Proc. seventh int. Congr. appl. Mech.* **4,** 194–208, 1948.

By means of charts designed on the basis of vector diagrams, behavior of a two-mass system in steady state of forced vibrations can be studied in a more general manner than is possible by other methods. Under certain circumstances, method might be applied to multimass system. From author's summary

79. Ludeke, Carl A., A method of equivalent linearization for nonlinear oscillatory systems with large nonlinearity, *J. appl. Phys.* **20,** 694–699, July 1949.

Systems with or without small viscous damping whose restoring forces consist of linear and cubic terms are considered. Approximate analytical expressions, coupled with experimental results lead to following conclusions: (1) Solution having same frequency as a sinusoidal forcing function is "practically" sinusoidal. (2) Frequency amplitude relation is essentially independent of amount of nonlinearity and damping except at resonance. (3) At resonance the amplitude frequency relation differs only by a factor from corresponding relation for linear systems. An experimentally determined graph of the factor in (3) is given.

Stephen H. Crandall, USA

80. Gonzalez Baz, Enriqueta, Relation between the parameter and the dimensions of the periodic solution of the van der Pol equation (in Spanish), *Comision Impulsora y Coordinadora de la Investigacion, Científica (Mexico),* Anuario 1947, 87–95, 1949.

The system of differential equations $3\dot{x} = 3y - \mu x^3 + 3\mu x, \dot{y} = -x$ is equivalent to van der Pol's equation. It has only one periodic solution which over a range of values of the parameter μ is close to a circle of radius k. The relation between k and μ

is computed explicitly by introducing polar coordinates and taking $r = k$ in the resulting expression for the derivative $dr/d\theta$. The relation is obtained from $\int dr = 0$ extended over $-\pi/2 < \theta < \pi/2$. The exact significance of k is not discussed; it is not clear, for example, whether k lies between the maximal and the minimal distance of the true solution from the origin.
Courtesy of Mathematical Reviews F. Bohnenblust, USA

81. Cartwright, M. L., **On nonlinear differential equations of the second order. III. The equation $\ddot{x} - k(1 - x^2)\dot{x} + x = p k\lambda \cos(\lambda t + \alpha)$, k small and λ near 1,** *Proc. Cambridge Philos. Soc.* **45**, 495–501, 1949.

By a combination of analysis and fixed point procedures, a rigorous proof is given of fact that the singular point in Poincaré plane of the pair of first-order differential equations obtained under the assumption of "slowly varying" functions (Appleton and van der Pol) gives an approximation to an actual periodic solution. Moreover, the periodic solution has the same type of stability as the corresponding singular point in the Poincaré plane. [The slowly varying functions are b_1 and b_2, where a solution of the form $b_1 \sin \lambda t + b_2 \cos \lambda t$ is assumed.]
Courtesy of Mathematical Reviews N. Levinson, USA

82. Rosenblatt, Alfredo, **On the phenomenon of subresonance for the van der Pol equation** (in Spanish), *Facultad de Ingeniería Montevideo. Publ. Inst. Mat. Estadística* **1**, no. 8 = *Bol. Fac. Ingen. Montevideo* **3**, no. 3, 12 pp., 1949.

Van der Pol's equation $\ddot{x} + x + \mu(A + Cx^2) = e \cos \omega t$, $A < 0$, $C > 0$ with a forcing term, is investigated for small μ when the frequency of the forcing term is an integral multiple of natural frequency of system, i.e., $\omega = n > 1$. Substitution of $x = x_0 + \mu x_1 + \mu^2 x_2^2 + \ldots$ in the equation determines the x_k successively, each up to an additive term of the form $a_k \sin t + b_k \cos t$. Can these constants be so determined as to make each x_k a periodic function with period 2π? For $n = 3$, answer is shown to be affirmative if and only if $-256A/C \geqq 7e^2$. For $n \neq 3$, the determinants of linear equations which determine the a_k, b_k vanish and construction breaks down.
Courtesy of Mathematical Reviews F. Bohnenblust, USA

83. Duncan, W. J., **The characteristics of systems which are nearly in a state of neutral static stability,** *Coll. Aeronaut. Cranfield. Rep.* **34**, 11 pp., 1950.

A method of successive approximations, requiring only solutions of linear systems at each stage, is derived for finding the time constant of a vibrating mechanical system such that the determinant of stiffness coefficients is nearly zero.
Courtesy of Mathematical Reviews P. Franklin, USA

84. Demontvignier, Marcel, and Lefevre, Paul, **A new harmonic method of study of stability of linear systems** (in French), *Rev. Gén. Electricité* **58**, 263–279, 1949.

An exposition is given of methods of testing stability of linear systems, from Routh to Nyquist. A simple method of constructing phase diagrams and of deducing stability from them, applicable to the majority of practical cases, is explained.
Courtesy of Mathematical Reviews P. Franklin, USA

85. Nardini, Renato, **On the stability of quasi-harmonic vibrations of a dissipative system** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **6**, 603–608, 1949.

Constant c is positive, function $w(t)$ is continuously differentiable, periodic with period T, has k relative maxima in a period and is positive. The solution $y = 0$ of $\ddot{y} + 2c\dot{y} + w^2(t)y = 0$ is called stable if every solution and its derivative tend to zero as $t \to \infty$. It is shown that stability occurs if $2c > \max |\dot{w}|/w$. Stability also occurs if $c < m$ and $2cT > k \log (M^2 - c^2)/(m^2 - c^2)$, where m, M are the minimum and the maximum of w.
Courtesy of Mathematical Reviews F. Bohnenblust, USA

86. Kopal, Zdeněk, **Radial oscillations of compressible gas spheres,** *Proc. nat. Acad. Sci. Wash.* **36**, 72–84, 1950.

Author points out that the four cases in which the problem of radial oscillations of compressible gas spheres have been solved analytically all belong mathematically to the same type. Solutions are characterized by possibility of expressing the eigenamplitude of radial oscillations in terms of hypergeometric series. Author sets out to investigate whether other models exist possessing this property. He first shows that for Fahrenheit values of central condention no other models than those already investigated exist for which an amplitude of small radial oscillations are of hypergeometric type. If no limit is imposed on central condention, he arrives at a whole new family of such models. In this new family, oscillations are found to be characterized by a partly discrete and partly continuous specter of frequencies and free oscillations. Author points out that this represents the first known instance of compressible gas spheres which can perform small radial oscillations in any frequency. He also points out that such models, if subject to a periodic distance, would respond to it by resonance independent of frequency, which might theoretically lead to a cataclysm.
Courtesy of Mathematical Reviews G. Randers, Norway

87. Baird, R. C., and Bechtold, I. C., **Mechanical vibration of piping induced by gas-pressure pulsations,** *Trans. Amer. Soc. mech. Engrs.* **71**, 989–995, Nov. 1949.

Authors present experimental curves of pressure pulsations in pipe lines of multicompressor systems and nearly duplicate these curves by combining electrical waves of different frequencies, thus indicating usefulness of analogies in studies of complex systems. Authors also describe three important means whereby the energy in such pressure pulsations is transformed into mechanical resonance vibrational energy in pipes.
Alexander Yorgiadis, USA

88. Levenson, Morris E., **Harmonic and subharmonic response for the Duffing equation $\ddot{x} + \alpha x + \beta x^3 = F \cos \omega t (\alpha > 0)$,** *J. appl. Phys.* **20**, 1045–1051, Nov. 1949.

Author classifies the periodic solutions of Duffing equation into four types: "ordinary harmonic," "ordinary subharmonic," "ultraharmonic," and "ultrasubharmonic." Considering the nondimensional form

$$\nu^2\xi''(\theta) + \xi + \delta\xi^3 = \cos\theta,\ \xi(0) = M$$

these correspond to the four types of solutions of the linear equation ($\delta = 0$) and reduce to these when $\delta \to 0$. (Existence proofs are omitted in paper.) Solutions for the four types are then obtained by a perturbation method, in which power series developments of ξ and ν in terms of δ are assumed, subject to proper initial and periodicity conditions. Thus, the forcing frequency is determined as a function of the initial value M (and parameters of equation). These approximate results are plotted as curves in the (ν^2, M) plane, for the various types of solutions, taking in each case $\delta = 0.1$ and $\delta = 0$ for comparison, where $\delta = \beta F^2/\alpha^3$. Author shows that no ordinary subharmonic solution exists whose frequency is exactly equal to free linear frequency.

A comparison is made between present solution of ordinary subharmonic type with those found by use of Rauscher's method. In this method the first approximation is the solution of the free

nonlinear equation ($F = 0$), with iterations performed in neighborhood of $F = 0$. The most significant result of comparison is excellent agreement for larger values of β than might have been anticipated. Certain discrepancies appear, for which author offers reasonable explanations.

P. S. Symonds, USA

89. Dreyfus-Graf, J., The dynamical vibrometer, theory and applications of a new electrical instrument to mechanical measurements (in French), *Schweiz. Arch.* **15,** 338–343, Nov. 1949.

Instrument is intended to measure dynamically: (1) Constant of elasticity of a clamped spring; (2) wobbling of a tooth (para dentose), fitting of a rivet, etc.; (3) mass or moment of inertia of a specimen; (4) coefficient of friction between dry or lubricated surfaces; and (5) viscosity of a liquid or a paste. Apparatus contains a vibration exciter and a receiver. Both consist of a moving-coil assembly with permanent magnets. Current through exciter coil determines amplitude of force, and may be kept constant. Voltage, generated in receiver coil, is a measure of amplitude of movement (supposing frequency is known). A spring connects exciter coil to receiver coil; latter can be fixed to object under observation. Apparatus contains a built-in calibration bar. Accuracy of measurements seems to be satisfactory.

R. G. Boiten, Holland

90. Geiger, J., The influence of twist on the natural frequencies and the direction of vibration of steam and gas turbine blades (in German), *Schw. Bauzig.* **68,** 3, 17–21; 4, 38–41, Jan. 21, 28, 1950.

Paper describes results of tests performed on five blades of rectangular cross section where ratio of the two sides varied from 2:1 to 20:1 and the top section was twisted with respect to the bottom a maximum of 45 deg. It was found that the first natural mode was changed very little by twisting blade, but second natural mode of twisted blade behaved very differently from that of untwisted blade. In this case, second natural frequency at first increased with slenderness of cross section until, at a 4:1 ratio, the increase was about 14% for a 45-deg twist. Then a sudden change took place and, at a 6.1:1 ratio, frequency decreased about 32% for a 45-deg twist. Mode shapes of twisted blades were naturally also quite different from those of corresponding untwisted blades for the higher modes.

N. O. Myklestad, USA

91. Kane, J. R., and McGoldrick, R. T., Longitudinal vibrations of marine propulsion-shafting systems, *Trans. Soc. nav. Archit. mar. Engrs.* no. 2, 40 pp., 1949.

Paper is concerned with excitation of the subject vibrations in ships by nonuniformity of wake in which propeller operates. It presents approximate methods for computing natural frequencies of vibration and magnitudes of thrust variation, damping, and amplitude of motion. Its chief value is that it contains experimental data obtained from several large modern ships that will be useful to naval vibration engineers.

Benjamin Smilg, USA

92. Federhofer, Karl, On the bending-torsion vibrations of circular rings with double-symmetrical cross sections (in German), *Oster. Akad. Wiss. Math.-Nat. Kl. S.-B. IIa* **157,** 299–320, 1950.

For a circular ring with a principal axis of the cross section in ring plane, the normal vibrations involving cross-section twist and bending normal to ring plane are uncoupled from other mode types. For this mode, author derives equations of motion for a ring with double-symmetrical cross section, and develops characteristic frequency equation for square cross-section case. One-dimensional bending and torsion theory is used, but with such effects as rotatory and warping inertia, and shear and warping energy taken into account. These secondary effects, which are conventionally omitted from analysis, permit author to show good analytic agreement with frequency measurements made by W. Kuhl [*Akust. Zeitschr.* **7,** p. 125, 1942] for a wide range of lower- and higher-order modes.

M. Goland, USA

93. Míshkis, A. D., Hystero-differential equations (in Russian), *Uspehi Matem. Nauk* (*N.S.*) **4,** 1(29), 190–193, 1949.

Author is interested in a type of functional equation of which a simple example is $y''(x) + f(x)y(x - g(x)) = 0$. In this note he outlines some results concerning the oscillation of the solutions. No proofs are given.

R. Bellman, USA

Wave Motion, Impact

(See also Rev. 12)

94. DeJuhasz, Kalman J., Graphical analysis of impact of bars stressed above the elastic range, *J. Franklin Inst.* **248:** 15–48; 113–142 (July and Aug. 1949).

Paper presents a graphical discussion with examples showing how solutions to impact problems of long bars may be found: first, involving only elastic stresses, and second, involving elastic and plastic stresses. Procedure is to set up two coordinate systems: (a) The t,x diagram, termed "location diagram," in which a point identifies an instant of time t and a cross section at distance x. (b) The p,v diagram, termed, "diagram of state," in which a point identifies "mechanical state," that is, coexisting values of velocity and stress. These two diagrams are sufficient for construction of stereograms which resolves into assigning a point in the vp to every point in the tx diagram.

Using these diagrams and given data of the material, velocity of propagation and wave factor are determined. The phenomenon from instant to instant and from point to point in the bar is followed, and thereby a stereogram of velocity and stress for bar is obtained. By integrating the stereogram of velocity, a stereogram of displacement may be constructed.

Author considers 16 problems which comprise impact at constant velocity of finite duration applied to one end of bar, lengths of bar being infinite and finite. In case of finite bars, the other than impact end may be free, fixed, or restrained. Impact stresses below and above yield stresses are considered. History and gradient of stress and velocity are shown in three-dimensional diagrams. Energy expended in impact and its distribution into plastic deformation and residual and elastic energies are determined, as well as residual permanent strain. Article includes a bibliography on impact of solid bodies.

Clayton O. Dohrenwend, USA

95. Adler, F. T., Sawyer, W. M., and Ferry, John D., Propagation of transverse waves in viscoelastic media, *J. appl. Phys.* **20,** 1036–1044, Nov. 1949.

Equations are derived for calculation of elastic constants of viscoelastic media from transverse wave propagation from a finite source plate in a rectangular cell, and representative numerical calculations have been made. In earlier investigations by authors and others, such calculations have been made under assumption of a plane disturbance in a medium of infinite extent.

New theory shows that finite dimensions of cell and source should have very little influence on wave length as determined from observations of strain double refraction, but attenuation

should be perceptably different from that of plane wave case. Experimental data are given, confirming these conclusions. However, the simple plane theory is shown to be adequate within present experimental error for majority of cases. Only in certain specified ranges of mechanical consistency will it be necessary to apply corrections taking in account sizes of cell and source. Good knowledge of higher mathematics is required to understand and apply the equations given in paper.

O. R. Wikander, USA

96. **Lowell, Sherman Cabot, The propagation of waves in shallow water,** *Comm. pure appl. Math.* **2,** 275–291, 1949.

Propagation of waves of small amplitude in shallow water is determined by a linear partial differential equation of hyperbolic type in variables x, y, t. Author first transforms this differential equation into an integral equation by Green's theorem and then derives equations for propagation of discontinuities in the surface elevation, imposing condition that discontinuous solutions have to be limiting cases of continuous solutions. One of these equations is equation of wave fronts, which is identical to the eiconal equation in geometrical optics. Second equation gives variation of magnitude of discontinuities along rays which are orthogonal to wave fronts. Methods for constructing wave fronts and rays are given. Author then shows that, for wave motions periodic in time, previous results for propagation of discontinuities can be used if wave length is very small. Last section of paper treats problem of oblique incidence of waves on a uniformly sloping beach.

H. S. Tsien, USA

97. **Kennard, E. H., Generation of surface waves by a moving partition,** *Quart. appl. Math.* **7,** 303–312, Oct. 1949.

Paper analyzes propagation of a small disturbance, generated by motion of vertical boundary in a semi-infinite, two-dimensional body of an incompressible fluid with a free surface. Case of infinite depth is treated in some (mathematical) detail, after which solution for finite depth is constructed by an extension of former solution. Particular attention is given to case of harmonic excitation, and both transient and steady-state regimes are treated. At large distances from boundary, traveling waves are found to be asymptotic to usual gravity waves found when free surface is infinite. Author shows that steady state at any point is effectively attained when $\omega t \gg \omega^2 x/g = 2\pi x/\lambda$ where ω is exciting (angular) frequency, x is distance from the boundary, g is acceleration due to gravity, and λ is wave length of traveling waves.

John W. Miles, USA

98. **Hartman, Philip, A characterization of the spectra of one-dimensional wave equations,** *Amer. J. Math.* **71,** 915–920, 1949.

The equation (*) $(px')' + (q + \lambda)x = 0$ is considered, where $p(t) > 0$ and $q(t)$ are both real and continuous, $0 \leqq t < \infty$. The limit point case is assumed and a boundary condition at $t = 0$, $x(0) \cos \alpha + x'(0) \sin \alpha = 0$. The spectrum is denoted by S. Let $x(t, \lambda)$ be a real solution and let $N(t, \lambda)$ be the number of zeros of $x(t, \lambda)$ on $0 \leqq t < T$. Let $\lambda' < \lambda''$ and let n denote lim inf $(N(T, \lambda'') - N(T, \lambda'))$ as $T \to \infty$. Then there are exactly n points of S satisfying $\lambda' < \lambda < \lambda''$ or $\lambda' \leqq \lambda < \lambda''$ according as (*) is oscillatory or nonoscillatory for $\lambda = \lambda''$. Moreover $n = \infty$ if and only if there are an infinite number of points of S in $\lambda' < \lambda < \lambda''$. In the proof it is shown that if $\mu_0(T)$, $\mu_1(T)$, . . . denote eigenvalues of the finite Sturm-Liouville problem on $0 \leqq t \leqq T$ with $x(T) = 0$ then λ belongs to S if, and only if, $d(T) \to 0$ as $T \to \infty$ where $d(T) = \min |\lambda - \mu_j(T)|$ for $j = 0, 1, 2, \ldots$.

Courtesy of Mathematical Reviews N. Levinson, USA

99. **Elianu, I. P., The Cauchy problem for the iterated wave equation** (in French), *Bull. Math. Soc. Roumaine Sci.* **48,** 102–144, 1947.

Author studies the iterated wave equation. Proceeding in a way analogous to that used in theory of polyharmonic functions, he obtains the fundamental solution and extends Green's formula. Then, using the idea of "finite part of an infinite integral" of Hadamard and some results of Nicolesco, he obtains solution of Cauchy's problem.

Courtesy of Mathematical Reviews L. Amerio, Italy

100. **Heinrich, G., Evaluation of impact measurements by means of the Laplace transformation** (in German), *Österreich. Ing.-Arch.* **3,** 327–336, 1949.

If temporal course of an acceleration is measured with an accelerometer of any type, the acceleration curve will generally not be quite correct. In case of temporally limited phenomena, a rectification of the curve is possible by using the damping curve that is obtained from the apparatus after a very short inelastic shock of fixed intensity. Influence of the latter is accounted for by an integral equation of the Volterra type, which can be solved by using the Laplace transformation. A graphical method is thereby developed which makes rectification possible.

Courtesy of Mathematical Reviews A. E. Heins, USA

101. **D'Auriac, Angles P., Influence of the elasticity of pipes and galleries on the celerity of water-hammer effect** (in French), *Houille blanche* **5,** 47–58, Jan.–Feb. 1950.

Paper develops a graphical method for determining velocity of propagation of water-hammer surges in thick pipes, from known elasticity characteristics of liquid and pipe. Inversely, method can be used to determine elasticity of pipe from measurements of velocity of propagation. Numerical examples are given to illustrate method.

Alexander Yorgiadis, USA

102. **Citrini, D., On the attenuation of a positive wave by action of a lateral overfall** (in Italian), *Energia Elett.* **26,** 589–599, Oct. 1949.

Author studies passage of a wave in a canal provided with a lateral spillway. If h_0 is depth of water in canal (steady flow), C level of spillway ($h_0 \leq C$), h_i and h_f heights of wave at both ends of canal, problem consists of calculating h_f as a function of h_i, the wave velocity $a = (q_0/h_0) \mp [1 + \frac{3}{4}(h_i - h_0)/h_0]$, the discharge q_0 (steady conditions), and ratio L/l of spillway length L to canal width l. Method used was first developed by Massau (1900) and more recently by Holtsters [*Revue générale de l'Hydraulique* no. 37 to 41, 1947]. Author follows the general lines of Holtsters' papers with few alterations, introducing boundary conditions given by spillway. He analyzes the variation of h_f/h_0 as a function of h_i/h_0, the Froude number for initial steady flow (q_0, h_0), and the ratio L/l.

Charles Jaeger, England

Elasticity Theory

(See also Revs. 144, 148, 217, 477)

103. **Marguerre, K., Strength problems of the engineer (Neuere Festigkeitsprobleme des Ingenieurs)** (in German), Berlin, Springer Verlag, 1950, 253 pp., 120 fig. DM 25.50.

Delayed since 1940 by World War II, this brief 1950 book presents useful treatment of mechanics of structural elements. It is divided into three principal parts devoted to strain measurement, theory of elasticity, and eigenvalue problems. The six chapters are by several authorities. Marguerre, the editor, provides chapter II on fundamental concepts of elasticity, and chap-

ter VI on buckling. Substantial portions of the material are taken from the excellent technical works on elasticity, vibrations, and shell theory by Biezeno and Grammel, Klotter, and Flügge, respectively.

Reviewer believes that this well-prepared Springer book is somewhat superfluous in view of "Technische Dynamik" by Biezeno and Grammel, "Statik und Dynamik der Schalen" by Flügge, and "Eigenwertprobleme" by Collatz, which are easily accessible in this country. W. H. Hoppmann, II, USA

104. Mann, E. H., An elastic theory of dislocations, *Proc. roy. Soc. Lond. Ser. A* **199,** 1058, 376–394, Nov. 1949.

The Volterra (1907) conception of small displacement "dislocations" associated with continuous stresses in a multiply connected body is extended to include cases where normal stresses parallel to dislocation plane are discontinuous. Elastic substances are isotropic with constant Young's modulus and Poisson's ratio throughout. Four solutions for plane and screw dislocations are given in cylindrical polar coordinates. The method is by separation of variables and also by complex variables. The stress solutions found are to apply in the elastic body except *at* or too *near* dislocation. The physical problem analyzed mathematically is that of finding stresses in elastic substance surrounding the slip planes of crystals. No attempt is made to estimate stresses *in* slip plane which is usually highly strained.

Reviewer notes that even in the orthodox theories on small deformations there is no restriction on finite magnitude of stress gradients. Therefore, in general, there can be an appreciable change in all stress values from one side of a slip plane to the other at a small but finite distance away. If, now, the slip plane of finite thickness is replaced by a discontinuity and adjacent barriers for mathematical convenience, there is no reason in the physical problem for author to retain some of the stresses continuous. Boundary stresses to act on elastic sides of slip plane must be found by analyzing the elastoplastic substance. Admittedly, this is difficult but appears to be one of outstanding problems for solution before there can be further progress in analyzing mathematically the elastic stresses around slip planes.
K. H. Swainger, England

105. Milne-Thomson, L. M., Plane elastic problems. I and II (in Spanish), *Revista Mat. Hisp.-Amer.* (4) **9,** 110–123, 141–153, 1949.

In the first part, author presents a general solution of the equations of plane elastic equilibrium, using the complex representation of Cartesian coordinates, and putting them in relation with the potential of forces acting in the plane. In the second part, various particular boundary shapes are considered, namely, the infinite plane, half plane, and circle. Also, the application of conformal transformation for passing from a circular contour to a more complicated form is mentioned. A large number of mistakes occur in the formulas, it being difficult to attribute all of these to the composer.
Courtesy of Mathematical Reviews B. Levi, South America

106. Arf, Cahit, On the existence of the solution of a problem of elasticity (in French), *Rev. Fac. Sci. Univ. Istanbul* (A) **14,** 75–85, 1949.

Author considers a certain system of nonlinear equations for real numbers $\beta_1, \ldots, \beta_{n-1}$ which arose out of a problem in plane elasticity discussed previously by him [same source, (A) **12,** 309–344, 1947]. It is stated that the system (7.12) of the previous paper is inexact, and the actual system of equations is given. This nonlinear system is shown to have a solution for any integer $n \gtrsim 2$. Examination of the cases $n = 2$ and $n = 3$ reveals that the solution is unique then, but an example is given, for $n = 4$, which has more than one solution.
Courtesy of Mathematical Reviews J. B. Diaz, USA

107. Fichera, Gaetano, On analytical problems of plane elasticity (in Italian), *Rend. Sem. Fac. Sci. Univ. Cagliari* 18 (1948), 1–22, 1949.

Author is concerned with existence and uniqueness of solution of boundary-value problems in plane elasticity. He formulates these problems in a fashion slightly more general than is customary, so that they shall contain as special cases classical problems of potential theory. Beginning his argument with reciprocity formulas and Green's formulas, he proceeds by analogy to methods of potential theory, treated from viewpoint of linear functional analysis. Thirty-three theorems are stated, and an indication of proof for some of them is supplied. Among them is one exhibiting a set of homogeneous polynomial solutions and another stating that these form a complete system.
C. Truesdell, USA

108. Roma, Maria Sofia, Integration of equations of three-dimensional elastostatic in an infinite cylinder (in Italian), *Ann. Scuola Norm. Super. Pisa* (3) **2,** 1948, 63–83, 1950.

Author studies the deformation of a three-dimensional elastic body, body forces being absent, which occupies the space between two coaxial right circular cylinders. Let Σ' and Σ'' be inner and outer cylinders, respectively. Three boundary-value problems are solved in first section: (1) Displacement prescribed on Σ' and Σ'', (2) normal stress prescribed on Σ' and Σ'', and (3) displacement prescribed on Σ' and normal stress prescribed on Σ''. Second section contains determination of displacement when there is only one cylinder present and it is subjected to axially symmetric pressure. Fourier transform method is employed in solution of all problems considered.
Courtesy of Mathematical Reviews J. B. Diaz, USA

109. Kuskov, A. M., The diffraction of elastic stable oscillations (in Russian), *Doklady Akad. Nauk SSSR* (N.S.) **70,** 197–200, 1950.

The three-dimensional boundary-value problem of dynamical elasticity for an infinite homogeneous isotropic medium exterior to a closed surface S on which the stresses are given, is equivalent to the determination of a scalar potential function and a vector potential function, each of which satisfies the wave equation where the two constants are different, and each expressed in terms of Lamé's constants and the frequency. An appropriate radiation condition is assumed and the system is reduced to the solution of a system of singular Fredholm integral equations of the type solved by S. G. Mihlin [*Uspehi Matem. Nauk* (N.S.) **3,** 3(25), 29–112, 1948].
Courtesy of Mathematical Reviews C. G. Maple, USA

110. Swainger, K. H., Non-coaxiality of principal normal stresses and the "strain" ellipsoid in the classical theory on infinitesimal deformation, *Nature* **165,** 159–160, 1950.

Apparently, author's intention is to point out that, while in an isotropically elastic body the principal axes of stress coincide with Eulerian strain axes, these in turn do not coincide with Lagrangian strain axes, but differ by an infinitesimal rotation. This fact is a discovery of Cauchy. Author's consequent objection to the classical infinitesimal theory is ill-taken because one of the assumptions of that theory is that local rotation is negligible. Author's claim that "just an infinitesimal 'rotation of the body as a whole' (in my sense) through angle θ leads to loss of one-to-one correspondence between principal normal stresses and

strains' defined by 'spatial-displacement' gradients" is not proved and, in the reviewer's opinion, is false.

C. Truesdell, USA

111. **Andelic, Tatomir P., Foundations of the mechanics of continuous media (Osnovi mehanike neprekidnih sredina** (in Serbian), **Belgrade, Nauchna Knjiga, 1950, vii + 231 pp.**

Book is primarily an exposition of mathematical foundations of theory of elasticity and fluid mechanics. Emphasis is on derivation of fundamental equations and general theorems, almost no consideration being given to problems associated with special geometrical configurations. Book is divided as follows: Introductory chapter on dyadics, affinors, and tensors; chapter I. Strain and stress in continuous media; chapter II. Theory of elasticity; chapter III. Hydromechanics.

Courtesy of Mathematical Reviews J. V. Wehausen, USA

112. **Elliott, H. A., Axial symmetric stress distributions in aeolotropic hexagonal crystals. The problem of the plane and related problems,** *Proc. Camb. phil. Soc.* **45,** 621–630, Oct. 1949.

Method of Hankel transformation, which has been found to be convenient in solution of "the problem of the plane" and related problems in case of axial symmetry for isotropic materials, is now developed for similar cases in anisotropic or aerolotropic materials. It is shown that solutions for problems of indentation of hexagonal plane by rigid punches can be found for punches of arbitrary axial symmetric shape. Solutions are given in full for cases of spherical, conical, and circularly cylindrical punches. Same method is used to find solutions for a material containing disk-shaped cracks between hexagonal planes. Results for isotropic case are shown to be directly reducible from general solution. It is pointed out that similar problems may be solved by methods of Green.

C. T. Wang, USA

113. **Muchnikov, V. M., On forces arising in coupling devices of a heterogeneous train during the starting process** (in Russian), *Doklady Akad. Nauk SSSR* (*N.S.*) **68,** 6, 1001–1004, Oct. 1949.

A mathematical investigation is given of the forces in couplings of a train consisting of two different types of cars, all heavy cars being located in one part of the total train length, and all light cars in the other. Tractive force of locomotive is assumed constant during the starting process. It is shown, in particular, that if the heavy cars are located at end of train, forces in couplings of front part at certain instances may exceed the tractive force of locomotive by as much as 20 per cent and, at other instances, may change sign, i.e., become compressive. A solution is briefly sketched for case when locomotive force is not constant but varies with time.

George Winter, USA

114. **Arzhanikh, I. S., Integral equations of the theory of elasticity** (in Russian), *Uspehi Matem. Nauk* (*N.S.*) **4,** no. 5(33), 176–177, 1949.

Under consideration are the surface and volume vector potentials:

$$v(q) = \int_S\int L(q, p)\bar{\sigma}(p)dS_p,\quad U(q) = \int_T\int\int L(q, p)\bar{x}(p)dT_p$$

$$w(q) = \int_S\int K(q, p)\rho(p)dS_p,\quad E_u(q) = \int_T\int\int D(\bar{u})dT_p$$

L, K are certain tensors; the vector $D(\bar{u})$ is determined by the component tensors of deformation. It is assumed that the closed surface S satisfies the Liapounoff conditions of "smoothness." It is indicated that certain systems of integral equations are to be solved for each of the four classical problems of elasticity (the first and second, interior, and exterior problems). The solution of any of these systems enables determination of the corresponding vector of deformation.

Courtesy of Mathematical Reviews W. J. Trjitzinsky, USA

115. **Maccaferri, Luisa, The formula for the volume variation in hereditary elasticity** (in Italian), *Atti semin. mat. fis. Univ.* **3,** 46–49, 1948–1949.

In Volterra's accumulative theory of elasticity the stress $t_j^i(t)$ and the infinitesimal strain $e_j^i(t)$ at time t are related by

$$(*)\ t_j^i(t) = \lambda\delta_j^i e_k^k(t) + 2\mu e_j^i(t) + {}_0\int^t [\phi(t,\tau)\delta_j^i e_k^k(\tau) + 2\psi(t,\tau)e_j^i(\tau)]d\tau.$$

By solving a Volterra integral equation and employing Green's theorem, author shows that the volume change Δv for a body V bounded by a surface S and subject to extraneous force F and surface force R is given by

$$(3\lambda + 2\mu)\Delta v = K(t) + {}_0\int^t k(t, d\tau)K(\tau)d\tau,$$

where

$$K(\tau) = {}_V\int \rho F(\tau)\cdot r dV + {}_S\int R(\tau)\cdot r dS,$$

$k(t, \tau)$ is the resolvent kernel for $3\phi + 2\psi$, r is the radius vector with respect to any origin, and the volume and surface integrals are to be carried out over the body at time t, whatever the value of τ.

C. Truesdell, USA

116. **Golushkevich, S. S., The plane problem of the theory of the limiting equilibrium of friable media (Ploskaya Zadacha Teorii predelnogo ravnovensiya sipucheĭ sredĭ)** (in Russian), OGIZ, 1948, 148 pp. (4 plates).

117. **Filonenko-Borodic, M. M., Theory of elasticity (Teoriya uprugosti), 3d ed.** (in Russian), Moscow-Leningrad, OGIZ, 1947, **300 pp.**

A considerably revised and rewritten version of a book originally issued in 1932 [2d ed., 1933] and intended as an introduction to elasticity theory for students in technical schools and universities. Chapter headings: I, Theory of stress. II, Geometrical theory of strain. III, The generalized Hooke's law. IV, Solution of the displacement problem of the theory of elasticity. V, Solution of the stress problem of the theory of elasticity. VI, The plane problem in Cartesian coordinates. VII, The plane problem in polar coordinates. VIII, Torsion and bending of prismatic rods. IX, More general methods of solution of the problems of the theory of elasticity. X, The bending of flat plates.

Courtesy of Mathematical Reviews

118. **Gol'denblat, I. I., On a problem of the mechanics of finite deformations of continuous media** (in Russian), *Doklady Akad. Nauk SSSR* (*N.S.*) **70,** 973–976, 1950.

In an isotropically elastic body, the stress-strain relations are of the form $t = \psi_0 I + \psi_1 e + \psi_2 e^2$, where t is the ordinary stress matrix, e is any Eulerian strain matrix, and the coefficients ψ_i are functions of the invariants of e. In order for such a relation to be derivable from an elastic potential, the ψi must satisfy certain conditions of integrability. For a particular choice of e the author derives these conditions. He then considers case when the ψ_i and the strain invariants are known functions of two parameters and when an equation of state connecting these six quantities and the temperature is known. For this case he uses his conditions of integrability to obtain the coefficients ψ_i as functions of the invariants. He remarks that his method is

general, but the labor of extending it to a situation in which more than two parameters are required would be forbidding. As an example, he derives in this way the simple nonlinear stress-strain relationships which Murnaghan ["von Kármán Anniver. Vol.," pp. 121–136, Pasadena, Calif., 1941] obtained by taking the classical quadratic expression for the strain as exact rather than approximate in a particular strain measure.

C. Truesdell, USA

119. Brodeau, André, Anisotropy, symmetry and heterogeneity in elasticity (in French), *Publ. Sci. Tech. Ministère de l'Air, Paris*, no. 229, vii + 61 pp., 1949.

Author shows that for isotropic and some types of aelotropic elastic bodies the assumption that a displacement discontinuity across a surface where the elastic moduli are discontinuous is transversal, leads to conclusion that tangential stresses are zero. He regards this result as evidence that a junction of two physical bodies may not be visualized as a surface of discontinuity. He discusses also case when there is a narrow layer in which the elastic properties vary continuously but rapidly.

C. Truesdell, USA

120. Signorini, A., Finite thermoelastic transformations. II, *Ann. mat. pura Appl.* (4) **30**, 1–72, 1949.

Paper is latter half of a systematic exposition of author's researches in the theory of finite elastic strain [for part I see same source, (4) **22**, 33–143, 1943]. Chapter I contains an amplification of his earlier treatment of uniqueness (and, in certain cases, nonexistence) of solutions of general equations when all quantities, including loads, are expanded as formal power series in an arbitrary parameter. Chapter II deals with author's theory of second-degree elasticity, in which stresses are only quadratic functions of finite strain components and yet are exactly derivable from a strain energy. Most of the main results were obtained previously by author and by Tolotti in papers which are cited in present work. C. Truesdell, USA

121. Rozovskiĭ, M. I., Thermal stresses in presence of after effect (in Russian), *J. tekh. Phis.* **19**, 6, 696–710, June 1949.

Formula of Boltzmann-Volterra theory of after-working in case of thermal stresses is generalized. The integrodifferential equation for two-dimensional deformation state is obtained and, from the solution of two-dimensional problem with nonuniform temperature distribution in an ideal elastic body, it is shown how to obtain the solution of the same problem considering after-working. Corresponding results are obtained for the symmetrically heated ball and spherical shell and the thin plate with nonuniform temperature distribution.

From author's summary by M. D. Friedman, USA

122. Finkel'shteĭn, B. N., and Fastov, N. S., On the theory of relaxation phenomena in solid media (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **71**, 875–878, 1950.

The strain energy of an elastic body is expressed as a linear combination of the quadratic invariants of strain tensor and of a relaxation tensor, which is taken as a measure of the deviation from state of static equilibrium. From this, a generalized Hooke's law is derived in which stress is defined as a function of both strain and strain rate. The number of fundamental elastic constants required to characterize a homogeneous isotropic elastic body is now increased to four to include, in addition to Lamé constants, two new viscosity constants. Results are applied to case of a plane transverse elastic wave in an unbounded medium.

Courtesy of Mathematical Reviews H. I. Ansoff, USA

123. Kondo, Kazuo, The geometry of the perfect tension field. I (in Japanese), *J. Soc. appl. Mech. Japan* **2**, 3–4, 1949.

Definitions and basic concepts of a perfect tension field, due to H. Wagner, are generalized to include first, case of three-dimensional Euclidean space and, next, a Riemannian space of any number of dimensions (although practical cases appearing in theory of light structures and of membranes are not so general.) The two fundamental theorems due to Wagner are so modified as to suit the new concepts. They assert that: (1) Divergence of stress vector (not of tensor) vanishes in tension field; (2) any trajectory of stress vector is a geodesic of constraining Riemannian space. These theorems hold also for special cases: (a) of a membrane constrained in a definite curved surface (a constrained system), (b) of a free membrane in three-dimensional Euclidean space (a free system).

Courtesy of Mathematical Reviews A. Kawaguchi, Japan

124. Kondo, Kazuo, A proposal of a new theory concerning the yielding of materials based on Riemannian geometry (in Japanese), *J. Soc. appl. Mech. Japan* **2**, 29–31, 1949.

Principal features of proposed theory are: (1) Any three-dimensional elastic body in Euclidean space is considered to be bent at the yield-point into a curved state forming a Riemannian manifold. (2) Resistance to sudden appearance of the Riemannian state can be related to a certain property due to the fine structure of material. (3) Field equation of equilibrium of forces acting in direction normal to original flat manifold in enveloping space is established by an application of results of author's paper reviewed above. (4) Obtained equation is of same type as that in theory of buckling of a flat plate. (5) Sudden yielding of a material such as mild steel will be obtained for a characteristic value of this equation under suitable boundary conditions.

Courtesy of Mathematical Reviews A. Kawaguchi, Japan

Experimental Stress Analysis

(See also Revs. 103, 185)

125. Day, E. E., Effects of low temperatures on electric strain gages, *Trend. Eng. Univ. Wash.* **2**, 1, 10–15, 32, Jan. 1950.

A method is presented for measurement of strain gages at temperatures as low as −295 F. Loading is accomplished by a beam in uniform bending surrounded by a chamber filled with liquid oxygen. For several commercial types of strain gages, strain sensitivity factors were found to decrease with decreasing temperature, reaching a minimum in range −100 to −140 F and then increasing again for lower temperatures to −295 F. Strain sensitivity varied less than 3% from room temperature to −200 F, and from 6 to 8.5% over entire test temperature range.

S. S. Manson, USA

126. Physics in industry, The measurement of stress and strain in solids, London, Institute of Physics, x + 112 pp., 1948.

Collection of papers on experimental stress analysis covering primarily, wire-resistance strain gage, miscellaneous mechanical and electrical gages, photoelastic method, and x-ray method. In addition, use of wire-resistance gages in combination in measuring component loads, and a discussion of electrical circuits suitable for electrical strain gages are considered.

The papers concerned with wire-resistance strain gage are excellent. Many individual factors governing behavior of this type gage are considered in detail and evaluated in many instances. Recommendations are made to overcome the adverse effect of many of factors discussed. In second paper, theory on

use of wire-resistance strain gages in combination for measurement of component loads is nicely presented. Examples are given to illustrate application of this theory. Author of third paper presents data on comparison of sensitivities of resistance gage when d.c. and high-frequency bridge e.m.f. are used. Final paper discusses various circuits suitable for electric gage use. Fifth, sixth and seventh papers present a review of photoelastic method and photoelastic materials. An introduction to theory underlying the x-ray method of stress measurement, a description of necessary instrumentation, and a good discussion of experimental methods are given in ninth and tenth papers.

George H. Lee, USA

127. **Coutinho, A., Theory of an experimental method for determining stresses, not requiring an accurate knowledge of the modulus of elasticity** (in French), *Inter. Assoc. Bridge Struct. Engng. Publ.* **7**, 83–103, 1949.

Since Young's modulus for concrete (E_c) is variable and not easily found, stresses cannot accurately be computed from measured deformations. It is claimed that they may be determined by measurements on spherical or cylindrical metallic (solid or hollow) inclusions. Using known solutions for the state of stress in an inclusion placed in a homogeneous stress field, it is concluded that, provided $E_s/E_c > 4$, a given percentage uncertainty in E_c will result in a much smaller percentage uncertainty in stresses. Experimental possibility of making necessary measurements is discussed.

L. E. Goodman, USA

128. **Pirard, A., On a photoelastic procedure to determine stresses in parallel successive planes** (in French), *Rev. univ. Min.* **V**, 329–337, 1949.

Paper deals with separation of combined photoelastic effect produced by assembled plates which, as in case of riveted joints, successively affect path of light in polariscope. Author suggests covering joint with a piece of tracing paper to diffuse light in that zone, and depolarize it. After this operation, a polaroid film is introduced between the two plates, or a polaroid film and a quarter-wave film. Photoelastic image then records only isoclinics and isochromatics produced by plate near observer. By turning specimen 180° around a vertical axis, effect produced by other plate of joint is obtained. Author claims that method is simpler than to silver intersurface and use reflection polariscope. This fact, however, does not seem to be proved.

Reviewer would like to point out that isochromatics of assembled plates can also be determined by using materials of very low optical sensitivity for all plates but one, which should be of a material of high optical sensitivity. Isostatics in two plates can be determined by using brittle coatings on outside surfaces. This method has the disadvantage of introducing different loading conditions if the two plastics do not have same moduli of elasticity, but seems to be much simpler.

A. J. Durelli, USA

129. **Bokshtein, M. F., On the resolving power of a polariscope for photoelastic studies** (in Russian), *Zh. tekh. Fiz.* **19**, 1103–1106, Oct. 1949.

In the photoelastic picture, a region of stress concentration, especially near point of application of load, appears generally as a dark or white spot and it lacks resolution. Author shows that both of these effects are due to change of thickness resulting from steep gradient of stresses. Darkening or brightening of area is due to a refraction of beam (as in case of a prism); lack of resolution is brought about by fact that fringe gradient exceeds resolving power of polariscope. Author shows how both of these defects can be corrected.

D. Rosenthal, USA

130. **de Leiris, H., A new pneumatic amplification method and its applications in extensometry** (in French), *Proc. seventh int. Congr. appl. Mech.* **4**, 121–127, 1948.

A differential pneumatic amplification-type strain gage is discussed both from construction and behavior points of view. Amplification factors of from 370,000 to 270,000 are obtained with 0.2-mm diam nozzles. Larger diameter nozzles reduce sensitivity; 0.6-mm diam nozzles yield an amplification factor of about 80,000. Over small ranges of displacement, amplification factors are constant. For example, in an instrument having 0.3-mm diam nozzles, the linear range is over 4 microns and 900-mm water-pressure differential. Near linearity extends somewhat beyond this range.

George H. Lee, USA

131. **Greenough, G. B., Strain measurement by x-ray diffraction methods,** *Aero. Quart.* **1**, 211–224, Nov. 1949.

Paper is primarily to describe principles of x-ray diffraction to those unfamiliar with method. The honest evaluation of the technique and its limitations will also interest those who have had difficulties in employing conventional x-ray diffraction techniques for stress study.

By far the most important distinction brought out in survey is that x-ray diffraction techniques, if carefully applied, can yield a strain analysis on the atomic scale, but that this analysis does not necessarily represent the macroscopic strains present. The familiar phrase "x-ray stress analysis" is an optimistic term for present state of development of x-ray diffraction strain study.

Selective nature of x-ray diffraction technique by virtue of the ordered array of atoms in the crystallite and the monochromatic x-radiation is clearly presented, and value of this technique for study of atomic strains is evident. Conventional methods of measuring atomic spacing are described, aided by photographs and diagrams. Presence of elastic and plastic anisotropy in the crystals is described, and its effect on interpretation of x-ray strains discussed. Extensive research is still necessary to bring about a real correlation between atomic strains and macroscopic stresses, and x-ray diffraction technique is, by far, the most useful tool which can be used to perform the survey.

Anthony B. Marmo, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 76, 218)

132. **Lubahn, J. D., and Sachs, G., Bending of an ideal plastic metal,** *Trans Amer. Soc. mech. Engrs.* **72**, 201–208, 1950.

Taking into account the induced curvature and progressive changes in the cross-sectional shape, problem of fully plastic pure bending of a rectangular beam is solved for the very wide beam (plane strain) and the very narrow beam (plane stress). The material obeys an incremental or flow law with the von Mises yield condition, and does not work-harden. Plots are given of stress and strain distributions and the varying location of the neutral axis obtained by successive approximations.

D. C. Drucker, USA

133. **Duncan, W. J., Normalized orthogonal deflexion functions for beams,** *Aero. Res. Counc.*, R. & M. 2281, 23 pp., 1950.

Paper aims at drawing attention to advantages of normalized orthogonal displacement functions in treatment of problems on static deflection of elastic bodies. Such functions can always be constructed step by step from a basic set which satisfy geometric boundary conditions of the problem. Detailed discussion refers

to beams, and general normalized orthogonal polynomial functions are obtained for doubly built-in uniform beams. These are the simplest functions which satisfy the conditions, and each is expressible in terms of three Legendre polynomials. For convenience, they are separated into functions which are symmetric and antisymmetric, respectively, about midspan. Tables are given of first three symmetric and first two antisymmetric orthogonal functions, together with their first and second differential coefficients. Short double-entry tables giving the deflections caused by isolated normal loads and by isolated bending couples applied to doubly built-in uniform beams are also included.

From author's summary

134. Uflyand, Ya. S., Exact solution of the problem of bending of a prismatic bar for a class of asymmetrical cross sections (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **69**, 751–754, 1949.

A solution is given of Saint Venant's flexure problem for a prism whose cross section is formed by arcs of two intersecting circles. Problem is solved in bipolar coordinates by use of Fourier integrals. The torsion problem was solved by same technique in title source, **68**, 17–20, 1949, [Rev **3**, 234].

I. S. Sokolnikoff, USA

135. Dörr, J., The vibratory behavior of an infinitely long beam elastically supported (in German), *Ingen.-Arch.* **16**, 287–298, 1948.

Under usual assumptions of small elastic bending, author treats case of a concentrated transverse load traveling with constant velocity along an infinitely long uniform beam resting on an elastic foundation. Damping forces which are proportional to local deflection velocity are presumed to act on the beam. Author restricts attention to steady-state response of the beam system, which remains after initial transients have died away. By a straightforward application of Fourier integral theory, it is shown how steady-state response can be derived directly, without need to study initial transient motion. By introducing certain simplifications such as assumption of small damping, approximate and relatively simple solutions for problem are deduced. It is found that a critical velocity of traveling load exists, for which large beam responses are encountered; for this circumstance, the energy input to system on part of traveling load varies inversely as square root of damping coefficient. For speeds below this critical speed, energy input by traveling load varies directly with damping coefficient. For speeds higher than critical speed, it is found that, to the first order in the damping coefficient, energy input is independent of value of damping coefficient. Author points out the analogy between this behavior and that of flow of a compressible fluid about an obstacle (critical speed is analogous to sonic velocity). Paper concludes with brief discussion of manner in which the complete solution for problem, including transient motion, can be deduced. M. Goland, USA

136. Lorin, P., Investigation to determine the span ratios for continuous beams (in French), *Publ. int. Assn. Bridge Struct. Engng.* **9**, 299–319, Nov. 1949.

Author investigates the most favorable span ratios for beams with hinged girders (Gerber beams) of symmetrical form. He assumes that the permissible stresses in all the beams are everywhere fully utilized. Then, problem takes the following form: The span ratios of the different beam members are to be chosen so that the arithmetic sum of the absolute values of the various areas of the moment diagrams becomes a maximum. The investigation is carried out for a uniformly distributed load p and a life load p_1, and results are given in tables and graphs for different values of $m = p_1/p$. The calculation is of an elementary character and has been made for supporting constructions with an infinite number of similar elements and also for one single element.

Theodor Pöschl, Germany

137. Galin, L. A., The elastic-plastic torsion of prismatic bars, *Grad. Div. appl. Math. Brown Univ.*, Transl. All-T9, 22 pp., 1949.

See Rev **3**, 1097.

138. Freiberger, W., The uniform torsion of an incomplete tore, *Austral. J. Sci. Research Ser. A* **2**, 354–375, 1949.

Problem is of practical interest in connection with calculation of stresses in close-coiled helical springs. Author determines a stress system which is equivalent to a single external force acting along axis of tore. Problem reduces to determination of a single stress function which can be expressed in toroidal coordinates and satisfy the boundary conditions on free surface of tore. Stresses and displacements are evaluated in terms of dimensions of tore and compared with results of known approximate theories.

Courtesy of Mathematical Reviews D. L. Hall, USA

139. Reissner, Eric, Note on the problem of twisting of a circular ring sector, *Quart. appl. Math.* **7**, 342–347, Oct. 1949.

Author considers problem of twisting of a circular ring sector subjected to two equal and opposite forces P acting along axis of ring and in a direction perpendicular to plane of ring (which approximates loading of a helical spring). Cross section of ring is taken as hollow with thin walls. It is also assumed that all stress resultants of membrane theory vanish except the shear stress resultant S acting over the cross section of ring sector. From equilibrium and symmetry conditions, this resultant satisfies the condition $d(rS)/ds + Sdr/ds = 0$, where s is arc length and r radius from axis of sector. Integration gives $r^2S = C$. Using this equation and the condition that external load $P = \oint S \sin \phi ds$, author obtains $S = P/r^2 \oint (dz/r^2)$, where $dz = ds \sin \phi$ and integral is taken over entire cross section. This formula corresponds to Bredt's formula for shear stress in torsion theory of straight thin-walled tubes. Using stress-strain relations in cylindrical coordinates, on basis of above assumptions author shows that $K/P = \oint \frac{ds}{Gtr^3}\left[\oint \frac{dz}{r^2}\right]^{-2}$, where K = axial deflection per complete turn divided by 2π and G modulus of rigidity. This formula corresponds to Bredt's formula for angular twist of a thin-walled cylindrical tube.

Application of results is made to case of a tube with circular cross section, and a correction factor is obtained for calculating maximum stress. (This corresponds to correction factor commonly applied for ordinary helical springs.) Author also shows that effect of curvature and direct shear effects on deflection are small for circular tube. Results are also applied to derivation of stress and deflection formulas for thin-walled ring sectors having rectangular cross sections of constant thickness. Work is of interest in connection with design of helical springs wound from bars having hollow cross sections. A. M. Wahl, USA

140. Freihart, Georg, Thin web shear beams with nonparallel flanges (in German), *Bauplan. Bautech.* **3**, 10, 318–320, Oct. 1949.

After deriving the known elementary formula for tapered-beam web shears, assuming that bending moment is completely carried by flanges, some simple applications are worked out in detail.

F. J. Plantema, Holland

141. Flügge, W., and Marguerre, K., Warping-restraints in thin-walled rods (in German), *Ingen.-Arch.* **18**, 23–38, 1950.

Paper is a general analytic discussion of torsional warping-restraint problem in thin-walled, uniform cross-section rods of both open and closed construction. Particular emphasis is placed on pointing out the common basis, although very different character, of mechanism in the two types of structure. Rods with Z-section (flanges and web perpendicular) and symmetrical tubes of rectangular cross section, both with and without corner stringers, are dealt with in detail as examples.

M. Goland, USA

Plates, Disks, Shells, Membranes

(See also Revs. 103, 193)

142. **Dzhanelidze, G. Yu., Survey of the work published in the USSR on the theory of the bending of thick and thin plates,** *Amer. Math. Soc. Transl.* no. 6, 28 pp., 1950.

Translated from *Prikl. Mat. Mekh.* **12,** 109–128, 1948.

143. **Marguerre, K., On the stability of a cylindrical shell with variable curvature** (in German), *Federhofer-Girkmann-Festschrift,* Wien, Franz Deuticke, 57–82, 1950.

Buckling of an infinitely long axially loaded cylindrical elastic shell of variable curvature with stiffened longitudinal edges is treated as an eigenvalue problem of linear homogeneous differential equations. Airy stress function and normal deflection are dependent variables. Equations are reduced to ordinary type by assumption of sinusoidal waves of constant length in longitudinal direction. Separation is then achieved by Fourier analysis with respect to circumferential coordinate. Results indicate that a cylindrical shell of variable curvature buckles at lower stress than a circular cylindrical shell of the same mean curvature; i.e., places of small curvature weaken the shell more than places of large curvature strengthen it.

H. L. Langhaar, USA

144. **Conrad, Karl Leroy, Stress distribution due to hydrostatic pressure on a parabolic boundary,** *Iowa State Coll. J. Sci.* **23,** 397–404, 1949.

Author obtains a solution in closed form to problem of a semi-infinite isotropic cylinder with a parabolic boundary subjected to hydrostatic pressure, by using complex function theory method. Stress distributions are plotted for a specific example.

Courtesy of Mathematical Reviews H. D. Conway, USA

145. **Storchi, Edoardo, On indefinite equations of statics of membranes tended on generic surfaces** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **8,** 116–120, 1950.

Paper deals with equilibrium of membranes, which are regarded as two-dimensional Riemannian geometries over which a stress system is defined but no stress-strain relation is assumed. By employing isothermal coordinates and introducing a transformation, author reduces the equilibrium equations to the form

$$\begin{aligned} \partial A/\partial x + \partial B/\partial y &= (A + C)\partial\mu/\partial x, \\ \partial B/\partial x + \partial C/\partial y &= (A + C)\partial\mu/\partial y, \end{aligned} \qquad [*]$$

where A, B, C are proportional to stress components and μ is a given function. For case when the first stress invariant vanishes ($A + C = 0$), he obtains the general solution of [*] in a form closely analogous to Airy's solution for plane. Then he tries to solve system [*] in general by putting A, B, C equal to linear combinations of a single scalar and its first and second partial derivatives. He concludes that such a representation leads to a solution if and only if the membrane is of constant Gaussian curvature. Resulting solution is that obtained by Finzi [title source, (6) **19,** 578–584, 620–623, 1934; cf. also the author, *ibid.* (8) **7,** 1949, 227–231, 1950].

C. Truesdell, USA

146. **Kondo, Kazuo, On the statics of membranes** (in Japanese), *J. Soc. appl. Mech. Japan* **2,** 1–2, 1949.

Problems of a two-dimensional membrane covering the smooth surface of a curved body are extended to an arbitrary number of dimensions, and concept of a constraining manifold is introduced. The statics of such a state are connected with general geometrical properties of a subspace in a Riemannian space. Two principal results have been obtained as follows: (1) Condition of equilibrium is, here also, the vanishing of divergence of stress tensor in connection with constraining manifold; and (2) constraining force is given by an expression containing Euler-Schouten curvature tensor.

Courtesy of Mathematical Reviews A. Kawaguchi, Japan

147. **Grigoryan, D. M., A normal impact on an unbounded thin membrane** (in Russian), *Prikl. Mat. Mekh.* **13,** 277–284, 1949.

Author studies elastic and plastic deformations of a thin unbounded membrane under impact by vertex of a right circular cone whose axis is moving with uniform velocity normal to membrane. Equations of motion are integrated and results are tabulated.

Courtesy of Mathematical Reviews T. C. Doyle, USA

148. **Manson, S. S., Direct method of design and stress analysis of rotating discs with temperature gradient,** *Nat. adv. Comm. Aero. Rep.* 952, 14 pp., 1950.

See Rev **3,** 853.

149. **Clark, G. L., The problem of a rotating incompressible disk,** *Proc. Cambr. philos. Soc.* **45,** 405–410, 1949.

Paper continues author's previous work on the mechanics of continuous matter [*Proc. roy. Soc. Edinburgh.* Sect. A **62,** 434–441, 1949]. Taking stress tensor to be diagonal with elements $(p, 0, 0)$, $(p, p, 0)$, or (p, p, p), he assumes as stress-strain law for so-called incompressible matter (*) $p = \alpha c^2\Delta(1 + \Delta)^{-1}$, where α is a constant (unstrained density), c the velocity of light, and Δ the dilatation. On basis of equations derived from an energy tensor, he concludes that plane waves of dilatation are propagated with velocity c in a medium satisfying (*). He then investigates the external and internal fields of a rotating disk, and draws the conclusion that a disk formed of material satisfying (*) undergoes no change in radius when made to rotate.

J. L. Synge, Ireland

150. **Shank, M. E., and Wulff, John, Determination of boundary stresses during the compression of cylindrical powder compacts,** *J. Metals* **185,** 561–570, Sept. 1949.

In order to compress a powdered mass, it can be put under pressure in a thick-walled cylindrical tube, between two plungers. The hollow tube is then subjected to radial pressure, as well as tangential shear from inside. Authors divide the packing into single small cylindrical disks and determine the strength of the hollow cylinder by superpositioning the pressure and shear caused by the single disks. They use certain influence numbers, obtained experimentally. The tangential strain at the outside of the hollow cylinder was measured by means of strain-gages. Consideration is restricted to the symmetrical case, where plungers press from both sides in the same way. Experiments show that in order to obtain homogeneous density of the powder, it is

desirable that the inside wall and the plungers sliding on it be smooth. A further improvement is obtained by lubricating the wall, so friction between wall and plunger is diminished. The finer the powder, the smaller is the shear transmitted to the cylindrical walls. The radial pressure which the packing exerts on the hollow cylinder has its highest value at the middle of the packing, and decreases at the ends. The reverse applies to the shear. When constructing the cylinder, to prevent too large elastic radial extensions, it is desirable to place a thin-walled steel tube within a thick-walled steel-tube or cast iron pipe.

L. Föppl, Germany

151. Shanley, F. R., Simplified analysis of general instability of stiffened shells in pure bending, *J. aero. Sci.* **16,** 590–592, Oct. 1949.

Analyzed problem is determination of size of frames necessary to insure failure between frames in bending of stiffened shells—i.e., to prevent general instability. Based on photographs of general instability failures, a simplified model is adopted. The most heavily loaded stringer is considered equivalent to two hinged bars, each of which is simply supported at an end and rests upon a flexible frame (represented by a spring) at an intermediate point. (Nonlinear effects are assumed to be small and, if present, are included indirectly in final evaluation of coefficient.) On basis of this simple model, a coefficient is developed, value of which is correlated with occurrence or nonoccurrence of general instability by comparison with test data. Correlation does exist, indeed, although there is a rather wide transition zone in which either type of failure may occur. Reviewer is of opinion that there are good prospects that transition range might be narrowed by a more refined analysis.

S. B. Batdorf, USA

152. Ruppeneït, K. V., Compression of a cylinder between two rough rigid plates (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **72,** 247–250, 1950.

A cylindrical sample is compressed between two rough rigid plates under the assumption that plane sections perpendicular to generatrix remain plane. The general yield condition is written in the form $(\sigma_z - \sigma_r)^2 + 4\tau_{rz}{}^2 = 4k^2 \sin^2 (\{(\sigma_z + \sigma_r)/2k\} + H/r)$, where σ_z and σ_r are axial and radial stresses, respectively, and k and H are constants. Problem is solved for this condition as well as for a perfectly plastic material. It is pointed out that in latter case the uniaxial state of stress usually assumed in engineering approximations holds only when height of sample h satisfies the relation $h \geqq 2R_0 \cot \xi_0$, where R_0 is radius of sample and ξ_0 is given by $H = k(2\xi_0 - \sin 2\xi_0)$.

Courtesy of Mathematical Reviews H. I. Ansoff, USA

153. Kroll, Wilhelmina D., Instability in shear of simply supported square plates with reinforced hole, *J. Res. nat. Bur. Stands.* **43,** 5, 465–472, Nov. 1949.

This is a timely paper illustrating use of numerical methods of computation for complicated elasticity problems. Paper covers comparisons with a theoretical analysis for a plate without a hole, but does not contain experimental results. Comparisons made, however, are not inconsistent with what would be expected.

R. G. Sturm, USA

154. Dědič, O., State of stress of a square plate with a circular hole (in German), *Federhofer-Girkmann-Festschrift*, Wien, Franz Deuticke, 143–160, 1950.

Author obtains a solution, approximate within the classical plate theory, to problem of a square plate with a centric circular hole under a full uniformly distributed load. Plate is simply supported at its straight edges, interior circular boundary being free. The plate equation as well as the boundary conditions for circular edge are met rigorously, whereas those for the straight edges are satisfied approximately in sense of least squares following method of Trefftz. Bending moments along sections of symmetry and support reactions are evaluated numerically for three values of the characteristic shape parameter.

E. Sternberg, USA

155. Koepcke, W., Influence surfaces and internal forces of boundary-supported rectangular plates (in German), *Ingen.-Arch.* **18,** 106–138, 1950.

Author outlines procedure, and also provides extensive numerical tables to facilitate calculations in practice, for solution of deflection and stress problems for statically loaded rectangular plates with the four edges restrained in arbitrary combinations of simple and built-in supports. General manner of solution is as follows: (a) A particular solution is found for the nonhomogeneous differential equation for plate deflection, corresponding to actual plate loading but not necessarily satisfying the edge boundary conditions; (b) through addition of a biharmonic polynomial to solution (a), the conditions at the four plate corners are satisfied; and (c) each edge is then treated in succession, according to scheme: A solution for an unloaded plate is derived, with the actual boundary conditions of problem satisfied at all edges except the one in question; here the boundary conditions are arbitrary. This solution is added to (a) and (b) and the proper boundary condition at edge in question is imposed on total solution. In the general case, a solution of type (c) must be added for each edge.

By means of tables, author systematizes the procedure for introducing solution (c) into a particular problem. When the two edges adjacent to edge in question are simply supported, procedure for both symmetric and antisymmetric plate loadings is relatively direct. When this is not the case, because of the unavailability of a precise solution of type (c), further difficulties are encountered. For the latter circumstance the edge boundary conditions are not satisfied exactly, but are satisfied by a requirement derived from condition that the sum of the squares of all the boundary-condition errors shall be a minimum.

Two typical problems are worked out in detail to illustrate application of procedure.

M. Goland, USA

Buckling Problems

(See also Revs. 103, 143)

156. Ramboll, B., Moments in columns with beam load (in Danish), *Frandsen Anniv. Vol. Lab. Bygn. Tekn. Meddr.* no. 1, 91–99, 1950.

Maximum bending moment in a column, carrying compressive loads P at both ends and transverse loadings, is $M = M_0(P_E - \alpha P)/(P_E - P)$, where M_0 is bending moment, neglecting bending deflection; $P_E = \pi^2 EI/l^2$, Euler's load; α is a coefficient, depending on shape of M_0 diagram. Author derives formulas, tables, and diagrams for α, when M_0 is caused by: (1) Uniformly distributed transverse load on a column with hinged ends; (2) concentrated transverse load; and (3) moment loads, acting at both ends.

S. E. Kindem, Norway

157. Nylander, Henrik, Torsional and lateral buckling of eccentrically compressed I and T columns, *Trans. roy. Inst. Technol. Stockh.*, no. 28, 32 pp., 1949.

Paper deals with design of steel columns of monosymmetrical or bisymmetrical cross section submitted to an eccentric compressive loading acting in the web plane, under which circum-

stance the column is subjected to torsion while buckling in lateral direction. In section I, author obtains an expression for the critical load of perfectly straight and plane members by basing his assumptions on the fundamental equation which is a modification of an expression first obtained by F. Bleich ["Stahlhochbauten," Berlin, Anhang, 1933]. Section II deals with members having an initial deflection. Departing from accepted procedure, author shows how formulas for calculating load-bearing capacity of compressed members subjected to torsional and lateral buckling can be deduced by means of a method corresponding to that adopted for members in plane buckling and based on an assumed initial deflection of the member, including effect of eccentricity. Various types of cross sections are studied, and formulas are given for oblique loads acting in the web plane. Reviewer believes these afford a rational method of design and reduce stability problems to stress problems. Method for designing columns in compression and bending in web plane as well as lateral deflection and torsion is summarized in section III. The appendix gives a worked-out example and tables to simplify calculation of critical load for standard sections.

S. K. Ghaswala, India

158. **Van der Eb, W. J., Some special cases of buckling,** *Ingenieur 's Gravenhage* **61,** 0.81–85, Nov. 4, 1949.

Paper is a continuation of an earlier paper with same title [Rev 2, 996]. In analysis of a column, consisting of two beams connected by battens at equidistant intervals, the assumption of infinite rigidity is now dropped. (For correction of some inaccurate statements in the paper, see author's note.) Furthermore, detailed numerical data are presented for the buckling load of columns on equidistant and equal elastic supports; application to design of shell bridges is elucidated. W. T. Koiter, Holland

159. **Meier, J. H., Buckling of uniform and stepped columns** I, II, *Prod. Engng.* **20,** no. 10, 119–123, Oct. 1949; no. 11, 116–119, Nov. 1949.

Problem of buckling of a pin-ended column containing a single discontinuity in cross section is solved by a clever geometric construction which satisfies continuity conditions at the step. From this, buckling load of an equivalent column of constant cross section is found. Application of solution to case of a piston rod-cylinder assembly is given.

Reviewer notes that although the construction is based on an initially imperfect column for which buckling load can only be defined by the known Southwell method, buckling load of a stepped perfect column can be obtained directly from same construction.

George Gerard, USA

160. **Bijlaard, P. P., Stability of sandwich plates in combined shear and compression,** *J. aero. Sci.* **17,** 1, p. 63, Jan. 1950.

Note dealing with parabolic form of interaction curves for over-all instability. This form was found to exist for infinitely long plates under shear and longitudinal compression in Report S.346 [*N.L.L.*, Amsterdam]. It is confirmed by calculations according to author's approximate method [*Proc. Kon. Ned. Ak. v. Wet.* **50,** 1 and 2, 1947]. F. J. Plantema, Holland

161. **Norris, Charles B., Ericksen, Wilhelm S., March, H. W., Smith, C. B., and Boller, Kenneth H., Wrinkling of the facings of sandwich construction subjected to edgewise compression,** *For. Prod. Lab. Rep.* no. 1810, 68 pp., Nov. 1949.

Part I contains a mathematical analysis which makes use of the assumptions of Gough, Elam and de Bruyne and applies to orthotropic as well as to isotropic materials. Apart from wrinkling of initially flat sandwiches in elastic range, which case is compared with more accurate results of others, authors deal with failure when the facings have initial irregularities and when the facings are stressed beyond the proportional limit. Part II contains a comparison of test results with results obtained from mathematical analysis. Various types of failure are discussed, and it is concluded that variations in strength between core and facings and in core properties may result in marked variation in failure stress. When these properties were accurately known, tests agreed reasonably well with appropriately chosen theoretical curves. Suggested design criteria are given in part III.

F. J. Plantema, Holland

162. **Hopkins, H. G., Elastic stability of infinite strips,** *Proc. Cambr. philos. Soc.* **45,** 587–594, Oct. 1949.

Paper presents an elegant mathematical solution of problem of buckling of an infinitely long isotropic elastic plate with parallel, simply supported edges, the two edges being subjected to identical opposing normal loads in plane of plate. Load distribution on an edge is required to be piecewise continuous, zero at infinity, and absolutely integrable over the range $(-\infty, \infty)$. Analysis is limited to case in which there are no shear loads, and no net axial load in plate.

Load distribution on either edge is considered to be specified, except for a factor that is independent of lengthwise coordinate. Problem is to determine value of this factor that corresponds to buckling. Solution is shown to be an eigenvalue of a fourth-order linear ordinary differential equation, or, alternatively, an eigenvalue of a homogeneous Fredholm integral equation with an unsymmetrical kernel. Complete solutions are developed for several cases of concentrated applied loads, and for case in which load distribution on either edge is constant in a finite interval and zero outside of this interval. H. L. Langhaar, USA

163. **Cox, H. L., and Riddell, J. R., Buckling of a longitudinally stiffened flat panel,** *Aero. Quart.* **1,** 225–244, Nov. 1949.

Analysis, which is confined to elastic range, results in equations from which the required cross-sectional area and moment of inertia of longitudinal stiffeners may be determined so as to prevent buckling of stringers and plate as a whole, up to point where sheet begins to buckle between the stringers. In usual type of longitudinally stiffened panel, width is large compared with stringer dimensions; hence, it is customary to neglect effect of edge support and to treat panel as a wide column. For panels in which width is relatively small, this effect may be accounted for by present analysis, which follows method used by Timoshenko but results in a solution in closed form.

The interpretation of the equations can best be expressed in authors' own words: "Briefly, results show that the stringer has to fulfill two requirements: first, it should itself be stable in flexure up to required limit: and second, it should be stout enough to resist tendency of plate to drag it into a plate buckling mode. The first requirement for a given form of stringer section puts a limit on the ratio of depth of stringer web to thickness of the plating; the second puts a limit on the stringer cross section area or, in effect, on the stringer wall thickness."

F. R. Shanley, USA

164. **Alumyaè, N. A., Application of the generalized variational principle of Castigliano to the investigation of the buckling stage of thin-walled elastic shells** (in Russian), *Prikl. Mat. Mekh.* **14,** 93–98, Jan.–Feb. 1950.

Conditions of equilibrium under hydrostatic pressure are given according to earlier work of Mushtari, Vlasov, and Alumyaè for

the passage from a state free from bending moment to an equilibrium state under bending moment. A certain integral R is given, the variation δR of which is zero, if the moments M_{ij}, the arising supplementary stresses S_{ij}, and the normal component v of displacement are varied in such a manner that the equilibrium and boundary conditions are fulfilled. S_{ij} and M_{ij} fulfill the equilibrium conditions if they are represented by four stress functions φ, φ_1, φ_2, v, being computed by $\delta R = 0$, together with the boundary conditions and a supplementary condition. For a state infinitesimally neighboring the critical one, a certain integral R_c is zero, by which the critical state of stress T_c is computed with the aid of stress functions as a stationary value of the quotient of two double integrals. Examples show it is possible to give T_c an upper and a lower limit (see, e.g., the pressed beam squeezed on the ends). H. Schlechtweg, Germany

165. Alumyaè, N. A., A variational method for the investigation of thin elastic shells in the buckling stage (in Russian), *Prikl. Mat. Mekh.* **14,** 197–202, 1950.

Buckling of thin elastic shells is discussed under the condition of hydrostatic surface loading. A variational formula is presented in which the associated contour integral contains a variation of stress function instead of a variation of tangential component of deflection which is employed in Lagrange's formula. Choice of admissible functions is thus simplified. Results are applied to problem of buckling of a cylindrical strip.
Courtesy of Mathematical Reviews H. I. Ansoff, USA

166. Cicala, Placido, On the elastic behavior of a thin quasi-cylindrical wall (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **7,** 99–103, 1949.

An approximate solution for certain cases of elastic buckling of cylindrical shells is given; results are presented in diagrams. P. F. Neményi, USA

167. Lo, Hsu, Crate, Harold, and Schwartz, Edward B., Buckling of thin-walled cylinder under axial compression and internal pressure, *Nat. adv. Comm. Aero. tech. Note* 2021, 29 pp., Jan. 1950.

Tsien's assumption that a cylinder under axial compression buckles when strain energy of buckled cylinder equals strain energy of unbuckled cylinder with the same end shortening (*J. aero. Sci.*, Aug. 1942, p. 373) is extended to case of a cylinder under axial compression and internal pressure. Experiments on a single specimen which was buckled at one pressure, released and buckled at another pressure, etc., check the theoretical increase in buckling stress due to pressure but not the absolute theoretical stress. L. H. Donnell, USA

168. Alumyaè, N. A., On a formula for the critical stress of a momentless stressed state of thin-walled elastic shells (in Russian), *Prikl. Mat. Mekh.* **13,** 647–649, 1949.

Critical stress formula for (1) a spherical shell compressed by a distributed load, (2) a centrally compressed cylindrical shell, and (3) a centrally compressed conical shell, is known. Author derives a general critical stress formula for a shell of an arbitrary shape, which reduces to the above mentioned cases for suitable boundary conditions. Author points out that his results have mainly a qualitative character and, with the exception of simplest cases, cannot be used for exact computations.
Courtesy of Mathematical Reviews T. Leser, USA

169. Mushtari, H. M., Nonlinear theory of the equilibrium of the boundary zone of an elastic shell (in Russian), *Doklady Akad. Nauk SSSR* (*N.S.*) **69,** 511–513, 1949.

In this sequel to author's earlier paper [*Prikl. Mat. Mekh.* 13, 121–134, 1949; Rev **3,** 238] the nonlinear equilibrium equations (2.4) and (2.5) of that paper are replaced by two approximate equations obtained by assuming $\epsilon h^{-1/2} \ll 1$, where ϵ is maximum elongation, and $2h$ is ratio of thickness of the shell to its smallest linear dimension. It is shown that use of linearized equations in determining the edge effect leads to serious errors. I. S. Sokolnikoff, USA

170. Plass, H. J., Sinusoidal buckling of angle bars with bending loads as a plate problem—exact solution, *Stanford Res. tech. Rep.* no. 4, 16 pp., Oct. 1949.

Author deals with buckling of angle bars, i.e., bars whose cross sections are composed of two legs of same thickness, perpendicular to each other. Because of symmetry it is enough to examine the plate which belongs to one of the two legs. For such a right-angled plate, author assumes simple support along axis of bar and in the two boundaries vertical to them. Fourth boundary of plate, however, is completely free according to reality. Load is in plane of plate, with linear stress distribution along the two opposite sides of plate normal to bar axis. In a former work, author approximated the buckling load according to Rayleigh-Ritz. Now he gives an exact solution of problem. Comparison with approximate one shows how closely it approaches the exact solution. Various bar dimensions and special conditions are discussed. Ludwig Föppl, Germany

171. Chwalla, E., On the buckling stability of laterally loaded compression beams with simply symmetrical sections (in German), *Federhofer-Girkmann-Festschrift*, Wien, Franz Deuticke, 125–142, 1950.

Author states the differential equations for twisting and bending displacements. Boundary conditions are given in detail. He then resorts to the energy method and uses first variation of energy integral to obtain the fundamental energy equations. Next, he assumes series solutions in the displacements: $u = L\Sigma b_i f_i(\zeta)$; $\varphi = \Sigma c_j g_j(\zeta)$, which lead to Ritz's equations for elastic buckling problem and yield the determinant whose vanishing provides the buckling loads. Galerkin's equations are then deduced.

Solutions are indicated for several cases in which the displacements can be written in terms of sine functions. For one of these cases an explicit formula in closed form is developed for the fundamental buckling load.

Some recent papers on the subject are referenced, including those of Timoshenko [*Jour. Franklin Institute*, **239,** Jan.-June 1945] and of Goodier [*Cornell University*, no. 27/28, 1941–42 and *Trans. ASME*, Sept. 1942]. The recent literature on the subject is briefly discussed. W. H. Hoppmann II, USA

172. Kindem, Sverre E., Lateral buckling of beams, *Kong Norske Vid. Sel.* **21,** 1, 1–12, 1948.

Part one treats lateral buckling of a straight prismatic beam with a thin-walled, open cross section, symmetrical with respect to a vertical axis. By equating external work and strain energy in usual manner, a formula is derived for critical load. Only an approximate expression is given for angular displacement along beam. In part two, the formula is applied to some typical examples: simply supported beams with uniformly distributed load, and cantilever beams laterally supported at free end. Part three collects seven approximate formulas for critical beam loads, the beams being loaded in their planes of symmetry and subjected to lateral buckling. Numerical example is given. Th. Pöschl, Germany

173. Goodman, Stanley, **Elastic buckling of outstanding flanges clamped at one edge and reinforced by bulbs at other edge,** *Nat. adv. Comm. Aero. tech. Note* 1985, 25 pp., Oct. 1949.

Critical loads for a series of 10 flanges were computed by Rayleigh-Ritz method considering the flanges as thin plates simply supported along loaded sides, built-in along edge opposite bulb, and elastically built-in in bulb, considered as a beam. Some results were checked by integration of corresponding differential equation. Most efficient flange is found to have a width of $3.4(A_F)^{1/2}$, where A_F is cross-sectional area of flange.

Mario G. Salvadori, USA

174. Gossi, Alberto, **The calculation of frames made of very flexible members** (in Italian), *G. Gen. civ.* **87,** 7–8, 351–360, July–Aug. 1949; **87,** 9, 583–594, Nov. 1949.

Author considers a simple portal with clamped feet, carrying a symmetrical load. If the three bars are slender enough, beam-column action becomes noticeable, and at a certain load structure will become unstable. This critical load is determined for various cases and compared with experiments on models.

W. Flügge, USA

175. Hickman, William A., and Dow, Norris F., **Data on the compressive strength of 75S-T6 aluminum-alloy flat panels having small, thin, widely spaced, longitudinal extruded Z-section stiffeners,** *Nat. adv. Comm. Aero. tech. Note* 1978, 23 pp., Nov. 1949.

Report presents results of compressive tests performed on sheet-stringer panels having proportions in ranges generally identifiable with thick-skin construction. John E. Goldberg, USA

Joints and Joining Methods

176. Johansen, K. W., **Theory of timber connections,** *Publ. int. Assn. Bridge Struct. Engng.* **9,** 249–262, Nov. 1949.

The function of a timber connection is divided into (1) Dowel effect of bolt, which depends upon its resistance to bending and upon resistance of wood to crushing; (2) tensional effect of bolt, which depends upon degree to which it is tightened, extent to which this tension is maintained in service, and upon friction between abutting surfaces; and (3) effect of dog, which depends on its form and strength, together with resistance of wood to crushing.

Statement of theory is good. A relatively small number of test results are given, apparently made on selected specimens, hardly enough, in opinion of reviewer, to serve as adequate proof for working load formulas given in paper. M. F. Sayre, USA

177. Hechtman, Robert A., **Conditions affecting the slip of structural steel bolted joints,** *Trend Engng.* **1,** 12–18, Oct. 1949.

Results are presented of exploratory tests conducted to determine conditions affecting the slip of structural steel bolted joints. Two types of joints were tested in static tension. Each type consisted of a center plate and two side plates of ASTM-A7 steel fastened with bolts rough forged from high-strength structural steel and heat treated. One type of joint was fastened with a single bolt and other type with two bolts on a line transverse to longitudinal axis of specimen. Two series of tests were conducted. In one series, bolt tensions were varied, and in other, clearance in bolt holes was increased while bolt tensions remained constant. Load-slip relations for both series of tests are presented in form of graphs. Frank Baron, USA

Structures

(See also Revs. 151, 204, 378)

178. **Hemp, W. S., On the analysis of statically indeterminate structures,** *Aer. Res. Counc. Rep. Mem.*, R. & M. 2396, Nov. 1948, 7 pp., 1950.

Report develops a general method for analysis of statically indeterminate structures. It concerns both a rigorous demonstration of validity of methods, and recommendations for their successful practical application. Results obtained are closely related to theorems of Castigliano. Principal result is a generalization of the usual form of Castigliano's second theorem. Abstractions necessary for successful analysis of structures are considered. Formulas for special cases and recommendations for choice of unknown parameters are given. Relation of method to that of Castigliano is discussed and procedure for practical calculation is summarized. An example illustrating choice of the unknown parameters in the case of a rectangular tube (wing structure) is given. From author's summary

179. **Sutherland, Hale, and Bowman, Harry Lake, Structural theory. An introduction to the basic concepts and principles of structural theory relating to trusses, rigid frames, and space frameworks,** New York, John Wiley & Sons, Inc.; London, Chapman & Hall Ltd., 1950, 394 pp. $5.

Purpose of book is instruction of advanced undergraduate students in structural engineering. Coverage: statically determinate and statically indeterminate structures. Ed.

180. **Beer, H., A general method for the analysis of spatial frame systems** (in German), *Federhofer-Girkmann-Festschrift*, Wien, Franz Deuticke, 1–16, 1950.

Method of moment distribution is extended for analysis of spatial frame systems. A set of nine coefficients are determined from properties of each member of frame, these being analogous to distribution factor for two-dimensional case. Analysis is carried out for spatial frames with and without joint displacements.

C. C. Wan, USA

181. **Arutyunyan, N. Kh., On the investigation of statically indeterminate frames with supports shifting with time** (in Russian), *Prikl. mat. Mekh.* **13,** 5, 489–500, Sept./Oct. 1949.

Paper considers elastic behavior of a statically indeterminate frame structure supported by a saturated, compressible layer of soil mass resting on an impermeable, incompressible base. When structure is subjected to external loads, soil mass is compressed and stress distribution in frame is a function of amount of settlement of supports. Fundamental relation between reaction at support X and settlement S is given by condition that elastic displacement of frame support must be equal to settlement of soil mass.

Settlement of idealized foundation is due to change in water content in a unit volume of foundation. An integral equation is derived defining this settlement as a function of time in terms of consolidation coefficient and thickness of soil layer. This equation, together with elastic characteristics of frame, is used to derive following integral equation defining reaction X at a support: $X(t) = X(0) - \beta_0 \int^t X(t)K(t - \xi)d\xi$, where $X(0)$ is value of reaction X without settlement of supports, β is a constant which depends on characteristics of frame and soil mass, $K(t - \xi)$ equals $_i\Sigma^\infty 1,3,5\ldots \exp[-\lambda_i^2(t - \xi)]$, λ_i depends on consolidation coefficient and thickness of soil layer, and t is variable time. General problem of a frame structure indeterminate to the nth degree may be reduced to a solution of a system of integral equations.

The elegant mathematical treatment of problem presented by author is an interesting new approach to solution of a frame supported on a compressible foundation. In reviewer's opinion, however, it has little practical value, as idealized foundation assumed in problem rarely occurs in practice, and results of mathematical solution may differ greatly from actual behavior of structure.

B. Bresler, USA

182. Schibler, Willy, Stability of top chords of open bridges considering the plasticity of cross-girders (in German), *Publ. int. Assn. Bridge Struct. Engng.* **9,** 453–468, Nov. 1949.

Buckling strength of top chords of a through truss bridge is investigated. The top chords behave as struts supported laterally in an elastic medium, stiffness of which varies according to buckling direction. This problem of elastic stability is solved and a numerical example illustrates the theory.

From author's summary by G. V. R. Rao

183. Waltking, Friedrich-Wilhelm, Computation of pontoon bridges by members (in German), *Bautechnik* **26,** 289–292, Oct. 1949.

Author proposes a method for analysis of pontoon bridges which avoids necessity of solving lengthy simultaneous equations. Solution is analogous to that of a beam on elastic supports. Problem is simplified because of many hinges which exist in a pontoon bridge. Any loaded section of bridge between hinges can be considered as supported on elastic foundations consisting of the pontoons and the two hinges. Spring constant of pontoon can readily be computed from volume of pontoon and density of water. Spring constants of hinges are dependent upon relative flexibility of girders and pontoons of adjacent sections, and are determined by a step-by-step process.

Karl Arnstein, USA

184. Brameld, H. G., Stress analysis for a seven-span continuous girder bridge 1200 ft. long, *J. Instn. Engrs. Austral.* **21,** 165–174, Oct.-Nov. 1949.

Paper describes in detail the methods and tabulations used in calculation of stresses in the new Fitzroy bridge at Rockhampton, Australia. Author has developed the method of elastic weights (put forward by Mohr and Müller-Breslau) and adapted it to practical analysis of continuous beams. Reactions of interior supports are taken as the statically indeterminate unknowns. Their values are given by solving a system of six complete linear equations with the aid of a standard electric calculating machine. This method is claimed to be superior to Cross's and all other methods of analysis, owing to simplicity of the routine procedure. At end of paper, author takes up problem of lateral distribution of loads between the four main girders of bridge for five different cases of loading. Despite simplifications used, this problem entails solution of 20 simultaneous equations. Reviewer is not convinced that presented technique affords any progress over standard methods. Besides, he thinks that use of an approximate analysis, such as that of Leonhardt, would have considerably simplified the analysis of transverse distribution of loads without material loss of precision.

Ch. Massonnet, Belgium

185. Kammerer, M. A., The bridge of Saint-Sylvestre on the Lot, II: Photoelastic study (in French), *Tech. mod. Constr.* **4,** 11, 321–329, Nov. 1949.

A photoelastic test was conducted to determine distribution of stresses in a reinforced-concrete bridge recently built in France. Author used Plexiglas to scale down 1:100 the box-type continuous-beam bridge of three spans and variable moment of inertia. Normal stresses obtained from test checked well with those obtained theoretically. Test proves, however, that shear stresses near supports are 25% larger than those computed with the theory.

Author does not justify use of a material of such low optical sensitivity as Plexiglas.

A. J. Durelli, USA

186. Ashdown, A. J., Prismoidal thin-slab structures, *Concr. constr. Engng.* **XLIV,** 9, 279–285, Sept. 1949; **XLIV,** 10, 317–322, Oct. 1949.

A method for analyzing stresses in structures, such as coal bunkers or hipped roofs, built up of thin concrete slabs. Since such construction has certain advantages over alternative shell-type construction, analysis is timely. However, it is quite impossible to understand these articles without referring at least to the first of five earlier articles quoted. The footnote in which these are mentioned shows an error: March should read April. Usefulness of this article must depend on validity of inital assumptions. These, however, are not set out but may be found in first of earlier series mentioned above. They appear to depend only on elementary statics and beam theory. Whether simplifications which are made are justified, must depend on results of experimental work yet to be done, but this reviewer feels that, while they are probably justified for steeply sloping slabs, they may lead to important errors in other cases. Symbols are not defined in these two articles, and this is all the more serious since it appears that in places the conventional ones have not been used.

Frank A. Blakey, Australia

187. Williams, D., Strength of aeroplanes in relation to repeated loads, *Aero. Quart.* **1,** 4, 291–304, 1950.

A detailed discussion is presented on effect of repeated loads on aircraft structures from point of view of capacity of the structure to withstand such loads, and also of their frequencies of occurrence in the life of the aircraft. In a specific instance of the Lancaster bomber, author shows, on basis of statistics obtained by examination of 700 V-g slides, how one can compute number of repetitions of specified load levels which must be designed for when a given risk of a structural accident is to be taken. It is pointed out that a risk of one structural accident for 10^8 flying hours per aircraft is a reasonable safety requirement when other more serious air hazards are considered. On this basis, author arrives at the important tentative conclusion that an airplane stands in no danger from possibility of failure by repeated application of loads larger than half the ultimate because they do not occur frequently enough.

The formidable total of repetitions of loads of low amplitude, to be expected due to forced vibrations set up by engine or propeller, is also considered. In contrast to heavier aerodynamic loads, author points out that vibration stresses can be reduced at the source (e.g., by careful design of engine mountings) so that even if they are the most dangerous, they are also the most tractable for designer. The reassuring view is taken that, in highly redundant structures, the inherent scatter of fatigue life for different structural elements makes it likely that local failures, which can be detected by inspection at regular intervals, will occur rather than catastrophic structural failure from fatigue.

Author recommends that additional work be undertaken to obtain further data on behavior of structures and structural elements under repeated loading, and to obtain more statistical data from flight records so that loads can be predicted more accurately.

S. Levy, USA

188. Schürch, H., On statics of thin airplane wings (in German), *Thesis E.T.H. Zürich,* 62 pp., 1950.

Thesis presents results of an analytical and experimental in-

vestigation of stresses and deflections in a flat sandwich-type structure such as might be encountered in wings and tails of aircraft when subjected to torsional and bending loads. Rectangular, trapezoidal, sweptback, and delta plan-form surfaces have been included in this investigation.

Sandwich is assumed to be built up of two faces which carry both normal and shearing loads in plane of faces, separated by a core capable of carrying shearing loads normal to center plane. Faces are stiffened by an orthogonal system of stringers. Work is summarized in a series of curves in which variation along span of (a) angle of twist, (b) normal stresses, (c) shear stresses, (d) transverse curvature, (e) torsional moment shear component, and (f) torsional moment warping component have been investigated for a sandwich-type wing tapered in plan form and an idealized elevator for various types of loadings. Torsional moment shear and warping components have also been determined for case of a rectangular wing supported on a torsionally rigid fuselage and subjected to torsional loads. Variation of transverse curvature and deflection along span of a sweptback wing has also been included. Comparisons of measured and computed values of deflections, angles of twist and stresses are presented.

K. Arnstein, USA

189. **Goodey, W. J., Two-spar wing stress analysis,** *Aircr. Engng.* 21: 247, 287–292, Sept. 1949; 248, 313–319, Oct. 1949; 249, 358–362, Nov. 1949.

Stress distribution in a wing with flexible ribs spaced at finite intervals is considered. A method based on minimum strain energy is proposed for solving the case of a two-spar box beam with skin effective in shear only. Fundamental equations are derived and details of setting up and solving numerical equations are emphasized. Tests of a model wing are described, and test results are compared with theoretical solutions, showing good agreement between experiment and author's method. Previous publications dealing with the same problem fail to include a detailed treatment of the numerical method of solution, such as given in this paper. A similar treatment of the same problem appeared in a paper by H. F. Michielsen [*J. Aero. Sci.* **16**, 659–674, 1949].

B. Bresler, USA

190. **Tate, Manford B., Shear lag in tension panels and box beams,** *Iowa Eng. Exp. Station*, Eng. Rep. no. 3, 1950–1951, v + 189 pp.

Thorough-going treatment of shear-lag problem of interest to all structural engineers dealing with box beams covers development of theory, comparison with tests on two beams, and analysis of results. Procedure is more exact than typical methods covered by references in extensive bibliography, perhaps too exact and time-consuming for routine analysis of airplane wing structures.

Solutions cover shear-lag effects both before and after buckling of compression sheet. Large deflection theory, in conjunction with strain-energy methods, is used for the case of buckling, and expressions are included to simplify numerical application for five common loading conditions.

Details assisting one to understand and apply the method are treated clearly and completely in appendixes.

Joseph S. Newell, USA

191. **Stevens, P. E., Vibrations in multiple-story structures from earthquakes,** *Bull. seism. Soc. Amer.* **39**, 261–265, Oct. 1949.

Paper describes a method of determining motions in several stories of a building produced by harmonic vibrations of its foundations. Although method of analysis is correct, final accuracy of results depends on reliability of available data. Presented analysis applies only to vibrations caused by an impressed sinusoidal wave-form disturbance. As all earthquakes and earthquake shocks are not of this form, analysis becomes very academic and has therefore a restricted application.

S. K. Ghaswala, India

192. **Chambaud, M., Elastoplastic theory of bending in beams of reinforced concrete** (in French), *Inst. tech. Batim. Trav. Publ.* 101, 17 pp., Nov. 1949.

Author develops an elastoplastic method of calculation of bending of beams of reinforced concrete, similar to methods proposed by F. Stussi [*Publ. int. Ass. Bridge Struct. Engng.* **1**, p. 487, 1932] and R. Saliger [*Beton und Eisen* **14**, p. 226, 1935]. Most interesting part of paper is description of a series of tests on breaking strength in bending in beams of reinforced concrete. Author finds good agreement between theory and tests.

Aris Phillips, USA

193. **Favini, Aldo, Analysis of hyperbolic tanks in reinforced concrete** (in Italian), *G. Gen. civ.* **87**, 10, 515–533, Oct. 1949.

Extensive presentation of analysis based upon theory of thin shells with double curvature known from previous publications (Lossier, Bolle, Orabona, Girkmann, Panetti, Laponche, Dischinger, Timoshenko, Flügge) by using differential equations of elastic equilibrium with approximate solution developed by J. W. Geckeler.

J. J. Polivka, USA

194. **Armstrong, William Edward Iredale, Bond in prestressed concrete,** *J. Instn. civ. Engrs.* no. 1, 19–40, Nov. 1949.

Paper deals with pull-out tests on $^3/_{16}$-in. high-strength steel wires imbedded in 4 × 4-in. concrete prisms up to 8 ft in length, and in bending tests on 8 × 4-in. beams reinforced with same wires. Basic concrete mix is given as 1:1.875:2.625 with rapid curing Portland cement, water-cement ratio being 0.435. No detail other than place of origin of coarse and fine aggregate is given.

Author talks of a ripple in wire which he describes as a sinusoidal waver of the axis, of 12 to 15-in. period, and seems to think that this defect influenced some of results with respect to the bond. It was author's intention to provide information on: difference in performance of a prestressed beam and a non-prestressed one of identical section and reinforcement; tensile load in steel as compared to pull-out load for steel in a prism of similar concrete; and on several variables concerned in bond. Chosen steel stress was 173,200 psi, equal to 80% of stress at 0.2% strain. Some measurements may be found very useful, such as that of variation of wire diameter under load, giving a practical value of $^1/_4$ for the Poisson ratio, and retakes of some pull-out tests showing that after ten months a value consistently higher than the original one was obtained due to shrinkage and other causes; but reviewer feels that paper suffers from too narrow an approach and would have gained tremendously by inclusion of more data, such as: comparison of theoretical vs. measured stresses in both pull-out and flexure tests; method of holding prism in pull-out test; comparison not only between a nonprestressed and a 173,200 psi prestressed unit but also of units prestressed to different intensities; and more details on concrete mixes.

Robert Quintal, Canada

195. **Corlett, E. C. B., The application of basic functions to girder networks,** *Trans. N. E. Coast Instn. Engrs. Shipb.*, part I, 66, 51–80, Nov. 1949–50.

A strain energy method is described for calculating deflections, bending moments, vibrations, and buckling loads of girder net-

works. Particular feature of method is use of "basic functions" as components in deflection series, these functions being well suited to approximate deflection field of networks in which longitudinal and/or transverse members are completely or partly fixed. They are of the form $F(x) = T(\cosh ax - \cos ax) - (\sinh ax - \sin ax)$, T a constant, and have properties closely similar to sine functions.

Various applications to ship structures are described, and appended tables and graphs give further harmonic orders of the functions than previously calculated, together with certain derived integrals.

F. J. Plantema, Holland

196. Collinson, E. G., A note on the load transference on multi-wheel bogie under-carriages, *J. roy. aero. Soc.* **53,** 1009–1012, Oct. 1949.

When a landing gear has two wheels in tandem, application of brakes will cause a redistribution of reaction force on wheels due to friction between wheels and runway. Formulas are derived for distribution of reaction in such cases. The formulas should be useful in design of tires, structural design of landing gears, or structural design of runway pavements. Possibility of using compensating linkages to avoid redistribution of reaction is discussed.

Stanley U. Benscoter, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 94, 192, 234, 245)

197. Hodge, P. G., Jr., An introduction to the mathematical theory of perfectly plastic solids, Providence, R. I., Brown Univ., 1950, iii + 396 pp. Paper.

A comprehensive review of the mathematical theory of the distribution of stress and strain in plastically deformed bodies. Chapter titles are: 1, Basic concepts; 2, Struts and beams; 3, Torsion; 4, Plane strain and plane stress with axial symmetry; 5, Plane strain—general theory; 6, Problems in plane strain; 7, Discontinuous solutions in plane strain; 8, Plane stress; 9, Elastic-plastic problems; 10, Approximate solutions; 11, Extremum principles.

Treatment is clear and of the quality to be expected of a book from Professor Prager's school at Brown University. Particularly interesting is the chapter on discontinuous solutions, on which pioneer work has been done by Professor Prager and his collaborators. In a few cases, relatively recent work has not been taken into account, such as in the section on deformation in rolling where the time lag amounts occasionally to 5 years, and points cleared up long ago appear problematic.

Looking over subject matter treated, reviewer is impressed by one outstanding point of weakness in the present mathematical theory: the Prandtl-Reuss method of taking into account elastic deformations where these are not negligibly small compared with the plastic deformations. Method assumes that deformations are purely elastic up to a sharply defined point where yielding begins, and from then on the elastic deformations, calculated with usual elastic constants, are superposed to the plastic deformations obtained with the classical methods used for ideally plastic (nonhardening) materials. In relation to real materials, this method seems completely illusory and represents no approximation in general. Apart from mild steel in state where it has a sharp yield point (in which case the theory cannot be applied), the stress-strain curves of all metals bend over gradually from the initial elastic part to the plastic part which, for large deformations, can be considered as roughly horizontal and thus approximating the ideally plastic material of the mathematical theory. Whenever elastic deformations are not negligible, however, strains in the plastic parts of the body are, as a rule, relatively small and correspond to the steeply rising curved initial part of the stress-strain curve for which the horizontal line of the ideally plastic material is no approximation whatever. The case of elastic and plastic deformations of comparable magnitude, therefore, must be treated with the methods of nonlinear elasticity (at least if strain increases monotonically, with fixed principal axes; otherwise, presence of Bauschinger effect precludes any general method of treatment). The large amount of work done with the Prandtl-Reuss method appears to reviewer almost a waste of time; it is hardly of sufficient interest from the purely mathematical point of view, and the results, in the best case, are qualitative if applied to real materials; in the worst, they can be misleading.

The appearance of a new book on this subject raises again the difficult question of nomenclature. The contents, e.g., of Sokolovskiĭ's book "Theory of Plasticity," and of Schmid-Boas' "Plasticity of Crystals" are not in the relationship of theory and experiment. These books deal with different subjects; the second with physical laws of plastic deformation, the first with calculation of stress and strain in plastically distorted bodies with the assumption of some idealized law of deformation which may or may not represent an approximation to behavior of real materials. The common name for two very different subjects has been a major nuisance for years, and the time has come to remedy the situation. Since the spatial distribution of a physical quantity is traditionally called its "field," one could call the mathematical treatment of stress and strain distributions in deformed bodies the "theory of the plastic field." "Plasticity" is a physical property; "theory of plasticity," therefore, logically means the physical (molecular) theory of that property.

E. Orowan, England

198. Hill, Rodney, A comparative study of some variational principles in the theory of plasticity, *J. appl. Mech.* **17,** 64–66 1950.

Boundary-value problems for a rigid-plastic so-called Saint Venant-Mises material are considered which lead to straining throughout the body. Hill's principle of maximum rate of working is proved for the actual stress system compared with all stress systems satisfying the equilibrium equations and the yield limit which may vary in body. A generalization of Markov's principle which determines actual velocity distribution to give an absolute minimum for all incompressible flows satisfying the boundary conditions is also proved. It is shown that these principles give lower and upper bounds, respectively, for rate of work over the part of the boundary on which velocities are given. Hill's principle is extended to a more general stress-strain relation based on concept of plastic potential. Uniqueness of a solution which involves straining throughout body is derived from both principles.

E. H. Lee, USA

199. Oldroyd, J. G., Rectilinear flow of non-Bingham plastic solids and non-Newtonian viscous liquids. I, *Proc. Cambr. philos. Soc.* **45,** 595–611, Oct. 1949.

For materials whose stress is an isotropic function of rate of deformation when intensity of stress deviator exceeds a yield value θ, author considers flows whose velocity field is of the type $u = 0$, $v = 0$, $w = w(x, y)$. In a region of steady plastic or viscous flow of this sort when pressure is constant, dynamical equations reduce to div (η grad w) = 0, viscosity coefficient being a function only of rate of strain deviator. After referring these equations to a coordinate net consisting of curves of equal speed and their orthogonal trajectories, author defines a "natural" coordinate system of this type in terms of function $\tau = f(\gamma)$ con-

necting shearing stress and velocity gradient in a simple shearing motion, and reduces problem of finding all possible rectilinear flows for a given material to that of integrating the equation

$$[\gamma\tau'\tau^{-2}\xi_w]_w + [\tau\gamma'\gamma^{-2}\xi_W]_W = 0 \qquad [*]$$

where $\gamma = \gamma(\xi)$, $\tau = \tau(\xi)$ is a parametric form of above equation $\tau = f(\gamma)$, and w and W are natural coordinates. For a plastic body any curve w = const may be a boundary, any such curve along which $ds/dw = \infty$ is a possible yield surface, and there are certain regularity conditions on velocity field; for a fluid, results hold subject to fewer restrictions.

Author proposes to solve general equation by inverse methods, choosing functions $\gamma(\xi)$ and $\tau(\xi)$ in such a way that exact solutions are possible. As an example, he considers a material whose rheological equation is rather similar to that of a Bingham solid; an interesting particular solution represents an infinite mass forced longitudinally along a blunt semi-infinite keel-like boundary whose axial plane is the x-axis, and for which a certain x = const plane is a yield surface. In an appendix, author takes up general problem of solving [*] by inverse methods. First, he exhibits a class of materials (including ordinary viscous liquid) for which [*] can be reduced to Laplace's equation. Second, he finds all rheological equations for which [*] may be integrated by Ampère's method; result is a fourfold infinity of materials, including the one already employed as an example. Third, he constructs general solution of [*] for all but a double infinity of these materials. Results yield a wide class of materials enjoying quite various rheological properties, for which exact solutions of rectilinear flow problem are available. C. A. Truesdell, USA

200. **McCance, Sir Andrew, The plastic behaviour of solids,** *J. Iron Steel Inst.* **163**, part 3, 241–249, Nov. 1949.

The views of some workers that plastic behavior of solids can be formulated by an extension of elasticity theory is regarded by author as untenable. It is assumed that atoms in a metal can exist in two states: the normal state with a regular lattice, and cold worked state with a distorted lattice. With each of these states is associated a specific volume so that change in specific volume after cold work reflects the proportion of atoms which have changed from one state to the other. A general equation: $S = 2S_0e^{-b\cdot a}\sin h(b^2 - c^2)^{1/2}\cdot a$ is developed, in which "true" stress S is expressed as a function of contraction in area a, a constant S_0 and coefficients b and c. The terms are evaluated by curve fitting for a number of metals. It is concluded that work-hardening is due to an increase in contribution of surface tension forces arising from creation of new surfaces at slip planes. In terms of the coefficients of his general equation, author discusses brittle behavior and effects of temperature, pressure, and atomic structure on behavior of a number of metals, rock salt, marble, starch and rubber.

Based on assumption that creep is solely a result of diffusion processes, an equation is proposed for metals expressing creep life to rupture under any given stress. G. M. Sinclair, USA

201. **Sokolovskiĭ, V. V., Some problems of the theory of plasticity for a power-hardening material** (in Russian), *Prikl. Mat. Mekh.* **13**, 655–658, 1949.

The strain-hardening material considered in paper obeys a stress-strain law of the deformation type, octahedral shearing stress being proportional to a power of octahedral shear strain (hence term "power-hardening"). Following two problems are treated: Bending of a wedge by a force applied to its edge, and torsion of a tapering shaft of circular cross section.

W. Prager, USA

202. **Shapiro, G. S., On the integration by quadratures of the equations of the plane one-dimensional problem of the theory of plasticity taking account of the hardening of the material** (in Russian), *Prikl. Mat. Mekh.* **13**, 659–662, 1949.

Paper is concerned with axially symmetric problems of plane stress or plane strain. Assuming that incompressible strain-hardening material obeys a stress-strain law of deformation type, author shows that solution of problems under consideration can be reduced to quadratures. W. Prager, USA

203. **Allen, D. N. de G., and Southwell, Richard, Relaxation methods applied to engineering problems. XIV. Plastic straining in two-dimensional stress-systems,** *Philos. Trans. roy. Soc. Lond. Ser. A* **242**, 379–414, 1950.

Specific problems treated in paper by relaxation methods concern tensile specimens with semicircular and V notches. In each case, growth of plastic region is studied under assumption of plane strain or plane stress. In most cases, plastic regions originate at bottoms of notch and grow from there until they merge on center line of specimen. In case of specimen with semicircular notches in plane strain, however, new plastic regions start to develop from points on center line after plastic regions originating at bottoms of notches have grown to a certain size. To this reviewer's knowledge, the possibility of this type of behavior had been completely overlooked in the literature. W. Prager, USA

204. **Naef, R. A., Shrinkage and creep of reinforced concrete structures** (in German), *Schw. Bauztg.* 68, 4, 29–31, Jan. 28, 1950.

Effect of shrinkage and creep of concrete on stress distribution within reinforced sections subject to nonaxial compression, as well as on redundant moment of an elastically restrained beam, is investigated. It is shown that the phenomenon is governed by exponential functions of time, that total deformation of simply reinforced sections may attain a multiple of the elastic deformation depending on amount of reinforcement, and that redundant moment of creeping beam is smaller than that of elastic beam, for same elastic restraint. A. M. Freudenthal, USA

205. **Sully, A. H., Metallic creep and creep resistant alloys,** London: Butterworths Scientific Publications; New York: Interscience Publishers, Inc., 1949, xii + 278 pp., 138 figs. \$5.50.

With present demand for materials capable of satisfactory performance at ever-increasing elevated temperatures, this volume is very timely. The introduction (chapter I) briefly reviews the historical background of the subject and the elementary aspects of the engineering significance of creep. Chapter II, "Measurement of metallic creep," includes a review of types of testing machines such as constant load, deadweight apparatus, and some constant stress loading mechanisms as developed by Andrade and others. Various types of creep specimens are considered, followed by a discussion of temperature maintenance and control. Importance of careful temperature control is emphasized. Means of measuring extension is considered, including mechanical and optical systems.

Chapter III discusses "Characteristics of creep curves," including a review of work of Andrade on transient and quasi-viscous creep. Empirical relationships proposed by various investigators are reviewed. A discussion of application of creep data for design purposes follows. Here the creep limit or limiting creep stress is considered. Some attention is given to other aspects of creep phenomena such as tertiary creep and micro-creep.

"Characteristics of deformation of metals and physical theory

of creep" in chapter IV will be of special interest to those interested in theoretical aspects of the creep problem. A comprehensive review of deformation process is given, both for single crystal and polycrystalline bodies. Work of Hanson and Wheeler is discussed in detail with regard to changes in microstructure during creep at various temperatures and strain rates. Attention is then given to transient creep range as regards effects in crystalline structure and deformation theory. Dislocation theory of Taylor is discussed and statistical aspects of creep process briefly reviewed. Discrepancy between the dislocation process and formation of slip bands is considered, and work of Frank, in connection with reflection of dislocations from crystal boundaries, and that of Leschen, Carreker and Holloman with regard to nucleation of slip bands, are treated. Reviews of theories of Orowan, Mott and Nabarro, and Smith with regard to applying dislocation mechanism to transient creep process are given. The quasi-viscous (or secondary) aspects of the creep process are next discussed. Theories considered include those associated with amorphous grain boundaries, viscous flow due to lattice defects, and dislocations. Work of Eyring and others, in which creep is considered as a rate process, is then reviewed. Problems of fracture after prolonged creep, microcreep, and creep recovery make up balance of chapter.

Chapter V is devoted to metallurgical factors affecting creep. These include crystal size, recovery and recrystallization, effects of prior strain, soluble impurities and alloy additions, phase change, precipitation hardening and insoluble impurities, heat treatment, and finally service environment. The final chapters are devoted to creep properties of various metals and alloys. Chapter VI deals with ferrous alloys such as low-carbon steels, low-alloy steels, austenitic steels, cast iron, and recent alloys. Effects of various alloying agents and metallurgical conditions are discussed. Chapter VII reviews creep properties of nonferrous alloys such as tin, lead, zinc and their alloys, magnesium and aluminum base alloys, copper and its alloys, nickel, nickel-molybdenum, nickel-chromium and chromium and chromium-base alloys. Attention is given to properties for various temperatures and stresses, influence of alloys and impurities and application. Finally, chapter VIII is devoted to development of creep-resistant materials. Here emphasis is given to requirements necessary to develop a material specifically for high resistance to creep at extreme temperatures. This includes improvement of present alloys and consideration of new types of alloys. Metallic elements are considered on the basis of their melting point, and higher melting point metals are considered as possible creep-resistant materials, either as pure metals or in alloy form. Such metals are titanium, vanadium, chromium, zirconium, tantalum, niobium (columbium), molybdenum, and tungsten. The metallurgical factors needed for high creep resistance are pointed out, such as alloying additives, grain size, and heat treatment.

Criticisms concerning this book are few. More emphasis on application of creep data to engineering design would be pertinent. Supplementing this, a discussion of effects of combined stress on creep should not be omitted. Emphasis is given to British research which, of course, is natural since this is a British publication.

Louis F. Coffin, USA

206. Wood, W. A., and Rachinger, W. A., The mechanism of deformation in metals, with special reference to creep, *J. Inst. Metals* **76,** 237–253, Nov. 1949.

Basic experimental research was performed on mechanism of deformation of crystalline metals. Results indicate that at low temperature (tests were from 20 to 500 C) mechanism of deformation, for both slow and rapid deformation rates, is by formation of slip planes. At elevated temperatures, however, the evidence seems to show conclusively that slip planes do not form; instead, the original crystals break down into a "cellular" structure. These cells are thought to be as nearly perfect as original crystal, and are fragments of it. Tests indicate that crystals initially about 0.1 mm diam break into cells of from 10^{-3} mm to 7.10^{-3} mm diam after straining, larger diameters corresponding to higher temperatures.

At a constant strain rate there is a fairly definite "critical" temperature at which mechanism changes from cellular type to slip-plane type. This critical temperature increases as strain rate increases. Authors do not mention that their results are in at least qualitative agreement with those of Zener and Holloman.

A correlation was found between cell size and resistance to deformation, high resistance corresponding to small size. Transition to slip-plane deformation corresponds to a minimum "critical" size. At larger sizes, an equilibrium condition is reached after about 7 to 8% deformation. Beyond this point no further significant change occurs in cellular structure. This condition of constant cell size corresponds to constant resisting force at increasing deformation; hence, is case of secondary creep. Both resisting force and rate of strain hardening are shown to decrease as temperature increases. These lead to phenomenon of steady state creep.

Results show considerable promise and, in opinion of reviewer, justify continued study directed toward further clarification of creep problem. In particular, test conditions might approximate service conditions more closely.

All work was done on 99.98% pure aluminum. Authors infer results will apply to other metals, but give no evidence. Since all important results are qualitative, implication is probably justified.

All tests were conducted at a constant strain rate. Two rates were used: a "slow" rate of 0.1% per hr, and a "rapid" rate of 10% per min. Rapid rate was considered comparable with those of ordinary tensile testing; slow rate was chosen to be characteristic of secondary creep. It is regrettable that this deformation rate was not chosen more nearly in line with present-day design practice—say 0.001% to 0.00001% per hr.

All testing was done with a standardized strip specimen 0.036 in. thick by $^1/_2$ in. wide with a 2-in. gage length. Resisting forces were measured with Tuckerman extensometers on an elastic member in series. Deformation measurements were made with an x-ray back-reflection camera using cobalt $K\alpha$ radiation and a specimen-film distance of 10 cm. Principal experimental evidence is presented in the form of 18 x-ray diffraction pictures and 14 photomicrographs.

W. Kenneth Bodger, USA

207. Lubahn, J. D., Simultaneous aging and deformation in metals, *J. Metals* **7,** 702–709, Oct. 1949.

Author points out that influence of precipitation or aging which occurs simultaneously with deformation may be entirely different from that which occurs without deformation. Results of constant strain rate, constant load, and variable strain rate tensile tests on an aluminum alloy, 61S, are presented. Author concludes that aging during plastic deformation may cause (1) Discontinuous yielding, (2) "transient" flow following a change in strain rate, and (3) an inverse speed effect at temperatures where aging effect is greater than inherent strain rate effect.

M. J. Manjoine, USA

208. Fusfeld, H. I., New interpretation of the n**-power law in plastic deformation,** *J. appl. Phys.* **20,** 11, 1052–1055, Nov. 1949.

It is shown that if relation between uniaxial true stress and natural strain of a particular material in the plastic state can be

represented by the so-called n-power law $S = k\delta^n$, this relation can be transformed to form $S = S_o(e\delta/n)^n$ where S_o is the ultimate *engineering* stress and e is the base of natural logarithms. Various relations among the variables are discussed and illustrated.

Merit D. White, USA

209. **Hill, R., Plastic distortion of nonuniform sheets,** *Phil. Mag.* **40,** 309, 971–983, Oct. 1949.

Paper deals with deformations of a flat sheet under plane states of stress. Thickness of sheet may be nonuniform in stress-free state, or become so during the plastic deformation. The state of stress then may be considered as plane when the thickness does not too rapidly change in any direction in sheet plane. Stress and strain state is analyzed by means of characteristics of stress and velocity equations, and results are given for von Mises yield condition and Tresca maximum shear stress condition. The analysis employs the yielding curve and the envelope of the Mohr circles in the $(\sigma_2 + \sigma_1)/2$, $(\sigma_2 - \sigma_1)/2$ diagram. As an example, the expansion of a circular hole with initial radius a in an infinite plate under influence of an increasing pressure acting over edge of hole is calculated. Yielding begins at edge and then material becomes plastic within some radius c. First no thickening occurs until $c = 1.751a$, and at higher pressures thickening ends at a radius $r_0 = 0.571c$. These results are included in those obtained by G. I. Taylor (*Quart. J. Mech. appl. Math.* **1,** p. 103, 1948; REV. **2,** 41) for a hole starting from zero radius (apart from some numerical errors in Taylor's paper).

Albert Kochendörfer, Germany

210. **Assur, E. L., Davidenkov, N. N., and Terminasov, Y. S., Residual stresses by simple extension** (in Russian), *Zh. tekh. Fiz.* **19,** 1107–1118, Oct. 1949.

Change of diameter of Debye-Scherrer circles on plastic deformation, studied by Bollenrath, Hauk, and Osswald (1939) and by S. L. Smith and W. A. Wood (1941, 1944), was attributed by former to residual stresses in a thin surface layer due to its lower yield stress, and by latter to a general expansion of the lattice. Smith and Wood found that the effect did not disappear after etching off a surface layer (of unspecified thickness). By etching authors removed carefully measured thicknesses of surface layer of flat tensile specimens of 99.6% pure Al, and measured variation with depth of residual stresses mechanically (from bending after one-sided etching) and by x-rays [back reflection from the (511) plane]. They found that, contrary to results of Smith and Wood, residual stress disappeared when a layer of about 0.1 mm thickness was removed.

Reviewer's remark: G. B. Greenough shows in a simultaneous publication [*Proc. roy. Soc.* **A 197,** p. 556, July, 1949] that phenomenon has at least two causes. In addition to surface layer effect, there is, in general, a volume effect due to the circumstance that orientations of grains that contribute to a given x-ray reflection are such as to give rise to a nonvanishing (positive or negative) mean residual stress in these particular grains after plastic deformation.

E. Orowan, England

211. **Scott Blair, G. W., and Reiner, M., The rheological law underlying the Nutting equation,** *Appl. sci. Res.* **A 2,** 3, 225–234, 1950.

The Nutting equation is an empirical power-law relation between deformation and time under constant stress. Authors merely suggest that an additional parameter be included to allow for "instantaneous" strain; this, of course, is standard practice in fitting creep data for metals. By differentiating the Nutting equation, a so-called rheological law is obtained, involving stress rate, notwithstanding that the Nutting equation has only been shown to apply to deformation under constant stress.

R. Hill, England

212. **Oldroyd, J. G., On the formulation of rheological equations of state,** *Proc. roy. Soc., ser. A, M. & P. Sci.* **200,** 1063, 523–541, Feb. 1950.

The invariant forms of rheological equations of state for a homogeneous continuum, suitable for application to all conditions of motion and stress, are discussed. The right invariance properties can most readily be recognized if frame of reference is a coordinate system convected with material, but it is necessary to transform to a fixed frame of reference in order to solve the equations of state simultaneously with equations of continuity and of motion. An illustration is given of process of formulating equations of state suitable for universal application, based on noninvariant equations obtained from a simple experiment or structural theory. Anisotropic materials, and materials whose properties depend on previous rheological history, are included within scope of paper.

(From author's summary.)

Author considers, in particular, hypothetical incompressible elastico-viscous liquid in steady two-dimensional flow between two vertical coaxial cylinders rotating with different angular velocities. He shows that apparently trivial change in equations of state leads to difference in predicted behavior of material in bulk. Result can be used to distinguish experimentally between various possible generalized equations of state for particular material.

H. A. Lang, USA

213. **Sokolovskiĭ, V. V., One-dimensional nonstationary motion of a viscous-plastic medium** (in Russian), *Prikl. Mat. Mekh.* **13,** 623–632, 1949.

Plane shear waves (displacement component $u_y = u_y(x, t)$, $u_x = u_z = 0$) and cylindrical shear waves (displacement component $u_\theta = u_\theta(r, t)$, $u_r = u_z = 0$) are considered in a viscous plastic medium. The relation between shear stress τ and shear strain γ is $d\gamma/dt = 0$, $|\tau| \leq k$; $d\gamma/dt = (\kappa/\mu)(|\tau| - k)$, $|\tau| \geq k$, where $\kappa = \operatorname{sgn} \tau$, and k and μ are constants. Plane shear waves determine the diffusion equation, and solutions are given for cases of constant stress and constant velocity applied for a period to the surface $x = 0$ of a semi-infinite body, after which the boundary is freed. A pair of linear simultaneous partial differential equations are obtained for cylindrical shear waves, and the solution is given for a sudden rise in shear stress at the surface, $r = r_0$, of a cylindrical hole in an infinite body, followed by linear reduction of surface stress with time.

E. H. Lee, USA

214. **Levi, Franco, Generalization of the relation stress-deformation in elasto-viscous solids** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **6,** 3, 304–310, 1949.

Paper discusses a solid composed of fibers, some elastico-viscous and the others elastic, the fibers adhering to one another. Flow of such a solid under stress is described by an integro-differential equation, immediately reducible to a linear differential equation of second order.

J. L. Synge, Ireland

215. **Castiglia, Cesare, Effect of retarded elasticity on co-action states of viscous origin** (in Italian), *Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **6,** 311–314, 1949.

Using two different simple special laws for behavior of an elastico-viscous fiber, author solves the equation given by F. Levi in preceding paper.

J. L. Synge, Ireland

216. Freudenthal, Alfred M., Stress-analysis beyond the elastic range, *Proc. Soc. exp. Stress Anal.* **6,** 2, 131–140, 1949.

Author reviews more or less well-known behavior of materials having elastic and viscous or plastic strains, and illustrates a few cases by means of stress-strain curves in loading and unloading curves. Cases of a visco-elastic and of work-hardening material are mentioned in simple examples. Article contains only few mathematical developments. A. Nadai, USA

217. Kochetkov, A. M., On the propagation of elastic-viscous-plastic shear waves in a plate (in Russian), *Prikl. Mat. Mekh.* **14,** 203–208, 1950.

A circular hole is punched in an infinite thin plate. Punch is assumed rigid and plate is assumed to deform in shear only, shear strain being constant across thickness. The stress-strain expression used beyond the yield limit is of the flow type, rate of shear strain $\dot{\gamma}_{rz}$ being given by a linear combination of the quantity $\tau_{rz} - F(\gamma_{rz})$ and its derivative, where τ_{rz} is shear stress and $F(\gamma_{rz})$ is an empirical function. A solution is presented for the ideally plastic case in which $F(\gamma_{rz}) = \tau_s$, where τ_s is yield stress, as well as for case of linear hardening in which $F(\gamma_{rz}) = \tau_s[1 + (n^2 - 1)(\gamma_{rz}/\gamma_s - 1)]$, where γ_s is the yield strain and $1 < n < 2^{1/2}$. Graphs are plotted for ideally plastic case showing stress and velocity histories in plate as well as distribution of residual strain.
Courtesy of Mathematical Reviews H. I. Ansoff, USA

218. Galin, L. A., On the existence of a solution of the elastic-plastic problem of torsion of prismatic bars (in Russian), *Prikl. Mat. Mekh.* **13,** 650–654, 1949.

Problem of elastic-plastic torsion of a cylindrical bar can be reduced to following problem concerning the stress function: to determine a function $\psi(x, y)$ with continuous first derivatives which vanishes along the contour of the given cross section, satisfies $\psi_{xx} + \psi_{yy} = -c^2$ for a given c, and has $\psi_x^2 + \psi_y^2 < 1$ in part of the cross section, and $\psi_x^2 + \psi_y^2 = 1$ in the rest. While Nadai's known soap film and sand hill analogy makes plausible the existence of the function $\psi(x, y)$, present paper represents the first rigorous investigation of this existence problem. Generality of investigation is restricted by assumption that the elastic core nowhere extends to contour of cross section.

W. Prager, USA

219. Batdorf, S. B., The interpretation of biaxial-tension experiments involving constant stress ratios, *Nat. adv. Comm. Aero. tech. Note* 2029, 25 pp., 1950.

The slip theory of plastic flow [see Batdorf and Budianski, title source, no. 1871, 1949; REV **2,** 1264] is applied to prediction of stress-strain relations in biaxial tension from stress-strain curve for simple tension. Check with experiment is made for a stress-ratio of 2:1. Experimental results and prediction of slip theory lie between the predictions of maximum-shear and octahedral-shear theories, and slip theory gives the best agreement of these three. E. H. Lee, USA

220. McAdam, D. J., Jr., Influence of temperature on the stress-strain-energy relationship for copper and nickel-copper alloy, *J. Metals* **185,** 10, 727–740, Oct. 1949.

Author presents a study of previously published data to show that mechanical state depends not only on total strain but also on temperature during straining for tests of copper and monel made at temperatures ranging from room temperature to −188 C. He discusses influence of temperature on stress-strain-energy relationship for metals. To obtain the influence of temperature only, he proposes a method and presents curves for separating the other effects. M. J. Manjoine, USA

221. Bilby, B. A., Static models of dislocations, *J. Inst. Met.* **76,** part 6, 613–620, 1950.

Current physical theories of metal plasticity postulate the now familiar elementary lattice distortions called "dislocations." Present model, like Bragg's bubble raft, is designed for qualitative studies to circumvent tedious calculations. Atoms are represented by balls, interatomic bonds by flexible rods; thus, lattice is highly anisotropic elastically. Dislocations are introduced by removing certain bonds, producing relative slip of freed parts, and replacing bonds in new positions. Photographs, fully discussed, illustrate: Single straight "edge" (Taylor) and "screw" (Burgers) dislocations; pairs of dislocations; forces on dislocation due to surface, applied stress, solute atom, second dislocation; combinations of edge and screw dislocations; and arrays of dislocations in transition surfaces. Photographs, while of limited value as direct visual aids (reviewer visualizes three-dimensional distortions better without them), successfully fulfill stated function of illustrating scope and limitations of model.

William Fuller Brown, Jr., USA

222. Leibfried, Gunther, and Lucke, Kurt, On the stress-field of a dislocation (in German), *Z. Physik* **126,** 450–464, 1949.

Paper is well summarized by author's abstract: The stress field of a dislocation is described by various methods and the results are compared. In the first section, a solution based on theory of elasticity is given; in the second section, a simplified version and generalization of Peierls discussion is given which takes approximate account of the atomic structure. It turns out that these solutions practically agree. Finally, these solutions are compared to those given by Taylor.
Courtesy of Mathematical Reviews. R. Truell, USA

Failure, Mechanics of Solid State

223. Minamiõzi, K., and Okubo, H., A note on the notch effect of metals. *J. Franklin. Inst.* **249,** 1, 49–55, Jan. 1950.

Experimental results are given which show that endurance limit of round bars with small diametrical holes increases toward limit of that of the solid bar as the hole diameter approaches zero. Some previous theoretical work indicates that the stress concentration factor approaches 3 under these conditions. This previous theory assumed the material was homogeneous and isotropic. A new theory is developed in which mean value of shear stress, over an interval adjacent to hole and perpendicular to length of bar, is expressed in terms of length of this interval and other factors. From a stress concentration factor, determined from fatigue measurements, length of this interval can be calculated. Values of 0.0089, 0.0059, and 0.0051 in. respectively, are given for mild steel, a special steel, and a dural. It is assumed that these lengths are associated with crystallite size.

Irwin Vigness, USA

224. Bragg, Lawrence, Slip in metals, *Physica* 15, 83–91, Apr. 1949.

Author describes behavior of dynamical model of crystal structure represented by small bubbles (2 to less than 0.2 mm in size) which are blown by a fine nozzle immersed in a soap solution. When in motion, bubbles slide past each other without friction, but at rest take up equilibrium positions like atoms in a crystal. Measured forces between bubbles can be likened to atomic forces, for the curve of force between bubbles as function of distance between centers has same general shape as that for atoms.

Movement of dislocation and subsequent recrystallization could be studied on two-dimensional bubble models as well as deformations occurring under compressive loading. In addition, theoretical calculations were made to deduce the plastic behavior of bubble mass from forces between bubbles which could be checked closely with experiment.

While behavior of bubble mass seems to be similar to that of metal crystal, author believes that it does not explain the creation of dislocations, for example, in the interior of a perfect copper crystal where these are found to take place at very low strains.

Erich G. Thomsen, USA

225. **Grossman, N., and MacGregor, C. W., "Transition" temperatures under various amounts of plastic flow,** *Weld. Res. Suppl.* 14, 551–555, Nov. 1949.

Paper is one of a series based on study of low-temperature brittleness of ferritic steel as carried out at Massachusetts Institute of Technology. In this program, slow-bend tests are performed at controlled deflection rates on bars notched so as to produce a variety of constraints. By recording force-deflection diagram of test bar at various low temperatures, a "brittle transition temperature" can be defined as that temperature where no observable plastic flow occurs.

Present tests are carried out from four lots of hot-rolled mild steel plates using bars with .01 in. and $^1/_{16}$ in. radii at base of notch. Deflection at fracture vs. temperature are plotted and reveal a rather drastic variation in deflection as temperature is lowered. For example, there is a difference of about 100 F between point of no observable plastic flow and that where $^1/_2$ deg angular bend for fracture occurs. It is shown that notch geometry, as well as material properties, influence brittle transition temperature.

Louis F. Coffin, Jr., USA

Design Factors, Meaning of Material Tests

Ⓑ226. **Lipson, C., Noll, G. C., and Clock, L. S., Stress and strength of manufactured parts,** New York: McGraw-Hill Book Co., Inc., 1950, xvi + 259 pp. $4.50.

Book represents an important effort to make available to designers a large amount of research results reported in technical publications. Some of the problems have been simplified to give designer charts which are easy to use. Influence of stress gradient on so-called fatigue stress-concentration factor, for instance, is not taken into consideration. Statistical analysis is not mentioned either, although in some cases, as in Fig. 4, it would have helped the understanding considerably. On page 13, the geometric stress-concentration factor is defined in such a form that it cannot be used for bending or torsion, although these two types of loading are considered throughout the book.

Almost 200 charts give geometric stress-concentration factors, fatigue stress-concentration factors for annealed steels and for quenched steels, and allowable working stresses as a function of hardness for those steels. Charts will be very useful, but designer should use them with caution and discrimination. Almost 300 bibliographic references, listed chronologically in an appendix, allow designer to go to original sources.

A. J. Durelli, USA

Material Test Techniques

(See also Rev. 487)

227. **Cox, H. L., and Coleman, E. P., A note on repeated loading tests on components and complete structures,** *Roy. Aero. Soc.*, 1–10, Jan. 1950.

A different excitation method is described for resonance principle fatigue testing machines, which involves use of a slipping clutch in excitation system. This feature permits use of a small exciting motor, as friction of clutch may be pre-set so that clutch slips until motor speed approaches resonant speed of the system. Method has been successfully applied and has advantage of being applicable to structures of considerable flexibility as well as those which absorb large amounts of energy.

H. R. Neifert, USA

228. **Landwehr, R., and Brabert, G., Optical interference measurements on the bending of plates** (in German), *Ingen.-Arch.* 18, 1, 1–4, 1950.

Classical optical interference method is applied to obtain deflection patterns for thin plates, which are compared to deflections obtained by analytical means. Technique involves a monochromatic light source to one side which is reflected onto the plate by a semi-reflecting mirror. Interference is obtained by inserting wedge-shaped glass plate just over test plate such that its lower face is parallel with specimen, and its upper face is normal to incoming light beam. Sharp interference patterns are observed.

Glass plates were used throughout experiment. Three cases are considered: A circular plate resting freely on its edges, a circular plate resting freely on three-edge points at 120°-angles, and a square plate resting freely on its corners. In all cases, a central concentrated load was applied. Comparison with analytical expressions using textbook values of elastic constants gave errors no greater than 3%. In addition, by assuming values for Poisson's ratio, elastic moduli for glass plates could be obtained by this technique.

Louis F. Coffin, Jr., USA

229. **Maxwell, Bryce, and Rahm, L. F., Impact testing of plastics: Elimination of the toss factor,** *Amer. Soc. Test. Mat. Bull.* 161, 44–48, Oct. 1949.

"Toss factor" in an Izod test is defined as a measure of kinetic energy of broken portion of specimen; this energy forms part of total loss of energy recorded in test, and if Izod test is to be regarded as measuring energy required to initiate and propagate fracture of specimen, it is clearly desirable that effect of toss factor should be eliminated. In new machine devised by authors, this objective is achieved by a reversed form of Izod test, the specimen being in motion, rather than at rest, before impact; basic idea is that kinetic energy of specimen, when in motion initially, includes kinetic energy of broken part and this energy will, therefore, not appear as a reduction of velocity. Theory of machine is given and results of tests on a number of thermosetting and thermoplastic materials are presented. These results are claimed to agree better with observed service behavior of materials than results of conventional Izod tests.

R. M. Davies, Wales

230. **Soete, W., and Vancrombrugge, R., Resistance to fatigue of wires used in prestressed concrete** (in French), *Ann. Trav. publics Belg.* 102, 5, 513–533, Oct. 1949.

Authors attempt to determine endurance limit of high-strength steel wires used in prestressed concrete, between stress limits used in practice. Tests are made on 5 and 7-mm diam wires; minimum stress was chosen equal to ordinary prestressing stress, i.e., 75 kg/mm²; fatigue tests are limited to $2 \cdot 10^6$ cycles.

Paper describes machine used, which was a modified 30-ton Amsler pulsator. Pulsating stress is measured optically by means of Martens mirrors and electrically by SR4 strain gages. Fixing of specimens in machine has given much trouble, as wires always tend to break in grips because of stress concentration induced by these. Authors have tried five different systems of

grips, with circular or vee, smooth or toothed notches. Best system is found to be that with vee toothed notches. Tests with 5-mm diam wires have all given fractures in grips, under a maximum stress varying from 95 to 105 kg/mm², according to system of grips used. Tests on 7-mm diam wires are made on normal and heat-treated specimens, by using the best system of grips. Three specimens fractured in grips and 9 in middle of specimen under stresses from 100 to 105 kg/mm². According to this reviewer, these tests demonstrate that the weakest point in wires is the fastening itself; they raise the question of real resistance of these fastenings to fatigue loadings applied to prestressed beams, which should be cleared up experimentally.

Ch. Massonnet, Belgium

231. Hummel, Otto Hermann, On wear under sliding friction, its generation and testing (in German), *Z. Metallk.* **40**, 10, 365–371, Oct. 1949.

In his constructive review about various aspects of wear under sliding friction, author stresses that morphologically there are as many types of wear as there are types of metallic contact; the problem of wear testing would be simplified by the feature that the number of the latter types would essentially be restricted according to mechanical properties of material to be tested. Further, considerable thought is given to requirements to be fulfilled in order to ensure repeatability of wear tests; in this connection, it is rightly remarked that scatter of results is not always indicative of some shortcoming in test technique, as it may well be characteristic for certain types of wear.

On basis of requirements thus established, a new wear tester was developed, albeit along mainly conventional lines: a loaded, polished pin circumferentially rubs against a polished rotating ring sparsely supplied with oil. One of the few new features is found in the test procedure: interruptions of a test, such as have normally been applied by others for measuring trend of wear, are disfavored on grounds that the consequent discontinuities in heating of test pieces might cause disturbing effects (recrystallization); the tests are continued until either a steady state is reached or until seizure sets in.

H. Blok, Holland

232. Clark, D. S., and Wood, D. S., The time delay for the initiation of plastic deformation at rapidly applied constant stress, *Proc. Amer. Soc. Test. Mat.* **49**, 717–737, 1949.

Design and construction of a special rapid-load testing machine is described with which tensile loads may be applied to a specimen within 5 milliseconds and longer. Tests made on an annealed low-carbon steel are presented and discussed in detail. It is shown that a definite time delay is required for initiation of plastic deformation in this material and that this delay time depends upon applied stress. Experimental results obtained on five materials for which the stress-strain curve does not exhibit a definite yield point are briefly presented. No time delay for initiation of plastic deformation is observed for these materials. It is concluded that time delay is associated with yield point of mild steel.

From authors' summary

233. Müller, E. A. W., Nondestructive material testing by the magnetic-powder method: I, II (in German), *Arch. tech. Messen* **156**, 95–97, Aug. 1948; **159**, 8–11, Jan. 1949.

234. Kê, T'ing-Sui, and Ross, Marc, An apparatus for measurement of extremely high internal friction, *Rev. sci. Instrum.* **20**, 795–799, Nov. 1949.

This is an improved form of torsion apparatus in which internal friction is measured by determining angle by which strain lags behind stress in forced cyclic vibration. A periodic voltage of sawtooth wave form is applied to a moving coil galvanometer, in which the test specimen constitutes the suspension; deflection of this galvanometer is a measure of shear strain. As the experiments are conducted at low stress levels, current is a measure of shear stress and is indicated by a second galvanometer. Projection of both galvanometer images on a special scale, and reading of double the distance between stress and strain deflections at position of zero strain, allow determination of time separation and, hence, phase difference. Apparatus has the advantage that measurements can be made at a constant stress amplitude and frequency. Results of measurements on internal friction appear to compare favorably with torsion pendulum measurements using free decay method.

J. M. Robertson, USA

Mechanical Properties of Specific Materials

(See also Revs. 229, 232)

235. Baillie, I. L. G., Directionality of wrought light alloy materials, *J. roy. Aero. Soc.* 11–18, Jan. 1950.

The weight-saving use of higher working stresses in aircraft construction has given rise to unpredicted failures of components that can be ascribed to directionality of mechanical and plastic properties. Largest directional effect is shown by ultimate elongation. Value of 0.4 for ratio of transverse to longitudinal elongation is an average of results scattered from 0.1 to 0.7. Whereas for longitudinal loading the elongation may be sufficient to accommodate strain concentrations so that stress concentration controls the design, for transverse loadings the low elongation values make strain concentration the controlling design factor. Drawings often need to specify type of material and manner in which the part is to be extracted.

C. Zwikker, Netherlands

236. Pardue, T. E., Melchor, J. L., and Good, W. B., Energy losses and fracture of some metals resulting from a small number of cycles of strain, *Soc. exp. Stress Anal.* **7**, 27–29, 1950.

Observations are made on behavior of materials when subjected to repeated stresses which exceed their yield strength. Standard specimens of seven different material specifications were tested in an R. R. Moore fatigue machine (operating at 72 rpm) at loads which would produce failure in range below 10,000 stress reversals. Energy dissipated during life of each specimen is evaluated and correlated with nominal stress. For the several materials investigated, it is found that energy dissipated increases as applied load is decreased. These tests also indicate that relative ability of materials to absorb energy at equal loads is not always a measure of their ability to withstand repeated cycles of strain at that load.

Reviewer's comment: Use of stress-relieved specimens, or an evaluation of effect of residual stresses on ability to absorb energy, would probably show a closer correlation between energy absorbed and fatigue life for different materials tested at same stress level than indicated by authors.

H. R. Neifert, USA

237. Herschman, H. K., and Thomas, Carroll, Fatigue characteristics of electroformed sheets with and without iron backing, *J. Res. nat. Bur. Stands.* **43**, 5, 477–486, Nov. 1949.

Flexural fatigue tests are reported on specimens cut from composite plates of two types: (a) Electroformed sheets consisting of one layer each of iron and nickel, with in some cases a thin layer of chromium (.0003 in. to .0006 in. thick) electroplated on to the nickel layer; and (b) plates formed by bonding electroformed sheets of iron and nickel to open-hearth iron backs. Tests were

made with Krouse plate fatigue machines, producing unidirectional bending.

In tests (a) it was found that a thin coating of chromium caused a reduction in fatigue strength, with the greatest reduction corresponding to thickest layer. This harmful effect was partially removed by heating to 500 F. In tests (b) main interest was in comparing fatigue properties obtained with two types of bonding agents, namely, 50-50 tin-lead solder, and a plastic adhesive. It was found that fatigue limits of plates bonded by plastic adhesive were higher than those of plates joined by soldering. This was attributed to effects of porosity in solder layer and of corrosion. Test results are consistent with service data of bonded printing plates of similar types. P. S. Symonds, USA

238. **Tipper, C. F., The fracture of mild steel,** *Weld. Res. Suppl.* 15, 4, p. 183, Apr. 1950.
See Rev 3, 265.

239. **Hill, H. N., and Barker, R. S., Effect of open circular holes on tensile strength and elongation of sheet specimens,** *Nat. adv. Comm. Aero. tech. Note* 1974, Oct. 1949.

Relative effect of open circular holes on static tensile strength and elongation was studied for aluminum alloys 24S, Alclad 75S, and Alclad 14S, of several different tempers. Specimens with drilled and reamed holes having a diameter $^1/_3$ the width of the thin sheet stock, were tested with a single central hole, two staggered holes, or a special four-hole pattern. In practical cases, reduction in tensile strength resulting from the open circular holes did not exceed about 5% for Alclad 14S-T6, Alclad 75S-T6, 24S-T81, and 24S-T86. In case of 24S-T3, 24S-0 and Alclad 14S-T3, however, strength reductions may be slightly greater than 10%. Reductions in elongation caused by the open circular holes in sheet specimens were greatest for least ductile materials. T. J. Dolan, USA

240. **Polakowski, N. H., The compression test in relation to cold-rolling,** *J. Iron Steel Inst.* **163**, part 3, 250–276, Nov. 1949.

Paper is divided into two parts. Part I discusses in detail the static compression test wherein discrepancies in existing information are pointed out. Many of techniques used for obtaining the ideal compressive-flow stress curve are reviewed. Author then describes a "multistage" method of conducting compression tests in which the specimen is remachined following each 25% compression. This procedure results in an increased flow stress when compared with other methods. In fact, using this technique, an increase in friction of loading platens causes a lowering of resistance to deformation. This is contrary to most observations.

In part II, the initial critical shape of specimen is analyzed. Analysis is applied to cold-rolling process, wherein various cold-rolling experiments of other investigators are discussed in light of these observations. Louis F. Coffin, Jr., USA

241. **Boklen, Rudolf, On the relation between the derived hardness curve and tensile strength of metallic materials** (in German), *Z. Metallk.* **40**, 10, 372–374, Oct. 1949.

Author determined experimentally a conversion factor u_m between the Meyer end hardness and tensile strength of metals, and found it to be approximately 0.3. Same relation was found for Ludwig hardness number and tensile strength. Neither cold working, hot working, nor type of metal affected conversion factor. It was also found to be relatively independent of exponent n in "Meyer's law." Hardness curve referred to in title is one previously presented [title source, **35**, p. 148, 1943] and is used merely to support the discussion. Frank J. Mehringer, USA

242. **Yoffe, B. S., Application of the method of microhardness measurement to the solution of some physical problems** (in Russian), *Zh. tekh. Fiz.* **19**, 1089–1102, Oct. 1949.

Paper is a review of literature on work done in field of microhardness. Author discusses various methods used for determination of microhardness, type of indenters, amount of loading, etc. He concludes that existing methods are far from satisfactory and stresses need for further improvement.

Examining field of application, he devotes a great deal of attention to information obtained on properties of surface layers. Despite shortcomings inherent in microhardness determinations, he believes that results obtained so far point to fact that the surface layer of metals is weaker than inner layers. He seeks to correlate these results to results of investigations on distribution of residual stress in specimens deformed plastically in tension. For sake of completeness it should be mentioned that results to which author refers are not the only ones available in literature. Experiments carried out by Smith and Wood do not substantiate the correlation which author seeks to establish [*Proc. roy. Soc. Lond. Ser. A* **182**, 404–414]. Another piece of information on properties of metals which results of microhardness appear to supply, is influence of grain boundaries on hardness of grains in a polycrystalline material. Measurements taken on surface of individual grains show a gradual increase of hardness from center of grain toward boundary. Also, average hardness of smaller grains is greater than that of larger grains in aggregate. Paper contains 31 references of which 18 are of Russian origin.
D. Rosenthal, USA

243. **Pullen, W. J., Some mechanical properties of foamed polyvinyl formal for use as an elastic stabilizer in sandwich structures,** *Aer. Res. Counc. Rep. & Mem.*, R. & M. 2344, 8 pp., 1950.

Study has been made of two samples of Canadian foamed polyvinyl formal. Neither sample shows marked anisotropy. This material, or subsequent improved forms of it, is worth considering as a low density core material for use in the sandwich construction of aircraft. From author's summary

244. **Hillier, K. W., A method of measuring some dynamic elastic constants and its application to the study of high polymers,** *Proc. phys. Soc. Lond.* **62**, part II, 701–713, Nov. 1949.

An apparatus is described whereby the velocity and attenuation of longitudinal oscillations over the range 0 to 5 C and 500 cps and 30 kcps are measured.

Results obtained with polythene are calculated on basis of several theories. George Halsey, USA

245. **Kolsky, H., An investigation of the mechanical properties of materials at very high rates of loading,** *Proc. phys. Soc. Lond.* **62**, part II, 676–700, Nov. 1949.

Describes an adaption of the Davies modification [*Phil. Trans.* **A 240**, p. 375, 1948] of the Hopkinson pressure bar [*ibid.* **213**, 437, 1914] which allows measurement of stress-strain relations in thin wafer-like specimens subjected to load pulses of the order of 20 microseconds. Pressure pulses were produced by firing detonators attached to one end of the ballistically suspended bar composed of two rounds separated by specimen. Stresses and strains were deduced from photographic records of output of special radial and longitudinal condenser microphones as recorded on a two-beam oscilloscope. Dynamic stress-strain relations were obtained for polythene; natural rubber (it is stated that neoprene GN, Perbunan and GR-S are mechanically similar under given experimental conditions); plasticized polymethylmethacrylate (Perspex); annealed pure copper; and lead. Sources, histories,

and composition of materials are not described in detail. Stress-strain relations for polymeric materials exhibited relaxation effects. These were fitted to a modified Boltzmann-type memory function of the form: $A \exp [-(t - \tau)/\alpha]$. Fitted values of time constant were of the order of one-tenth the time of loading, i.e., α of the order of 1 microsecond. (Irwin, using quite different methods ["Brown University second symposium on plasticity," April, 1949, unpublished] finds time constants for conical indents into thick steel which are likewise approximately one-tenth the loading time of the indents, ca. 100 microseconds. Taylor [*J. Inst. civ. Engr.* **8**, 486, 1946] noted a similar relation for much longer loading pulses of the order of 10 milliseconds.)

Waller George, USA

246. Marin, Joseph, and Cuff, George, Creep-time relations for polystyrene under tension, bending, and torsion, *Proc. Amer. Soc. Test. Mat.* **49,** 1158–1180, 1949.

Results of tests at three loading conditions at 77 F were correlated by existing theory based on a power relation between stress and creep rate. Creep rates, calculated from tension data, for creep in bending and torsion were within 38 and 60%, respectively, of measured values. Reviewer feels that scatter in creep rates and narrow range of stresses employed may not justify selection of the empirical power relation as opposed to the theoretical exponential relation for creep rate vs. stress. This choice may, in part, account for disagreement reported above.

W. N. Findley, USA

247. Gohn, G. R., Cummings, J. D., and Ellis, W. C., The creep characteristics of compression molded polyethylene, *Proc. Amer. Soc. Test. Mat.* **49,** 1139–1157, 1949.

Constant load creep tests in tension at various stresses at room temperature are reported for polyethylene with and without carbon black and antioxidant. Aging was shown to improve creep resistance, and artificial weathering was shown to have no effect on polyethylene with carbon black and antioxidant.

W. N. Findley, USA

248. Reissinger, Siegmund, Determination of resistance to crack propagation in rubber (in German), *Arch. tech. Messen* no. 158, T120, Dec. 1948.

249. Washa, G. W., Scholar, C. H., Lewis, D. W., and Witey, N. H., Tests for air-entraining agents in cement and concrete, *Amer. Soc. Test. Mat. Bull* no. 163, 61–69, Jan. 1950.

Tests on concretes and mortars made at three laboratories showed that N-T air compared favorably with vinsol resin and darex in producing improvements in freezing and thawing resistance, placeability, plasticity, and homogeneity of concrete. Also, agent did not adversely affect setting time, rate of strength gain, volume change, or sulfate resistance. Paper adds considerable data to literature that has already been published showing advantages to be gained by controlled air entrainment, and also adds valuable information on technique of testing and interpretation of results of freezing-and-thawing tests.

From authors' summary

250. Kelly, J. W., and Haavik, Norman E., A simple field test for consistency of concrete, *Amer. Soc. Test. Mat. Bull.* no. 163, 70–74, Jan. 1950.

A simple field test for consistency of concrete is described which consists in observing penetration of a 30-lb. 6-in-diam metal ball into the surface. Through a coincidence, the penetration roughly equals half the slump. Similar penetration tests have recently been developed in England, Germany, and Spain. Results of laboratory and field tests are given; use, precision, and applicability of test are discussed.

From authors' summary

251. Clark, Arthur P., Bond of concrete reinforcing bars, *J. Amer. Concr. Inst.* **21,** 161–184, Nov. 1949. = *J. Res. nat. Bur. Stands.* **43,** 565–569, Dec. 1949.

Further studies were performed to compare the bond resistance of concrete reinforcing bars when tested in beams and pull-out specimens. Various types of deformed bars were tested and depth of concrete under bar, length of embedment, strength of concrete, and diameter of bar were varied. Slip of bar was measured at loaded and free ends. General behavior of bars was similar in the two types of tests. Test results confirmed that for good bond a maximum ratio of shearing to bearing areas of 5 to 6 is desirable for deformations. On basis of test results, revisions were proposed in tentative specifications for deformations of reinforcing steel. Bond strengths obtained with bars conforming to these revised specifications were much greater than those developed by bars in common use at the time ACI allowable bond stresses were established.

Andrew Brodsky, USA

252. Arutyunyan, N. Kh., The theory of an elastic stressed state of concrete, taking account of creep (in Russian), *Prikl. Mat. Mekh.* **13,** 609–622, 1949.

Author discusses deformations of an elastic continuum in presence of creep assuming a linear relation between stresses and the amount of creep they produce.

W. Prager, USA

253. Mhatre, S. H., and Preston, J. M., A new apparatus for measuring the density of fibres, *J. text. Inst. Proc.* 40, T831–T838, Dec. 1949.

A densitometer has been developed for measurement of fiber density in various solutes. From this, the sorption of solutes may be calculated. Conditioned and weighed fibers are packed in a glass basket to which a float is attached. Assembly is immersed in a chosen solute such as air-free water. A piece of iron wire within float serves as core of a solenoid which surrounds apparatus. Various pulls may be exerted on float by varying current through solenoid. By making due allowance for dry weight of fibers, weights and volumes of basket and float, and pull of solenoid, fiber density may be calculated. It is believed that method is accurate to 2 points in 100,000 when a 5-cc float is employed.

Rogers B. Finch, USA

254. Duke, C. Martin, Engineering properties of coral reef materials, *Proc. Amer. Soc. Test. Mat.* **49,** 964–976, 1949.

For engineering purposes, the tropical calcareous earth materials derived from coral reefs may be classified as coral, limestone, and cascajo, with coral the ancestor of the other two. Using this classification, a basis is suggested for the systematization of our knowledge of the engineering properties of coral reef materials. Lack of such a basis heretofore has been the cause of much confusion among engineers. The biologic and geologic aspects of coral reef formation are briefly summarized.

Results of tests on cascajo from Guam are presented. This important material exists plentifully on the older tropical islands. The test results, supported by experience, indicate excellent mechanical-, asphalt-, and cement-stabilization properties and fair concrete-making properties for properly selected materials.

The basic principles developed for design and construction with more conventional earth materials have been found to apply in practically full force to reef-derived materials. A bibliography

is given on origin and growth of coral reefs and on properties of reef-derived materials. From author's summary

255. **Schwertz, F. A., The structure of porous materials from gas penetration rates,** *J. appl. Phys.* **20,** 1070–1075, Nov. 1949.

Gas penetration rates have been used to study the structure of three commercial grades of sintered Pyrex glass. Specific surface areas, "average pore diameters," pore population densities, average particle diameters, and particle population densities have been calculated on basis of obtained data.

From author's summary

256. **Thorpe, P. L., Tremain, G. R., and Ridley, R. W., The mechanical properties of some wrought and cast aluminum alloys at elevated temperatures,** *J. Inst. Metals,* **77,** part 2, 111–140, 1950.

An investigation was conducted in the British National Physical Laboratory to provide data on commercial and experimental aluminum alloys used in construction of aircraft engines. Results are presented from tensile, fatigue, and creep tests at various temperatures in range 20 to 450 C for 17 wrought and 7 cast alloys. Among these materials were several experimental alloys developed during World War II. A nonstandard thermal strain test was also employed to measure the resistance to cracking of several cast alloys when subjected to heating and cooling cycles under axial restraint. Data were intended primarily to provide design data for selection of metals for use in pistons, impellers, and compressors subjected to repeated stressing at elevated temperatures. The large volume of test data is given in tables and charts. At 250 C there appeared to be no marked difference between fatigue properties of the wrought alloys. It was found in axial load fatigue tests that the superposition of a mean tensile stress produced only a slight reduction in the allowable alternating stress (as compared with tests employing zero mean stress). The alloy found to possess highest creep resistance at 250 C did not exhibit high tensile or fatigue strength. Though the alloys are designated by British code and specification numbers not familiar to most American readers, the results contain valuable detailed information on influence of a number of alloying constituents on mechanical properties.

T. J. Dolan, USA

257. **Chevigny, R., and Syre, R., Creep of aluminum alloys** (in French), *Rev. Metall.* **46,** 10, 682–687, Oct. 1949.

Creep test results on aluminum alloys are given to show that number of creep test specimens usually used may be reduced. Basis for this conclusion is that tests reported on aluminum alloys show that stresses to produce 0.002 in. per in. strain in 500 hours, stress that produces no creep strain after 500 hours, and yield stress using 0.002 in. per in. strain are all approximately equal.

Joseph Marin, USA

258. **Greenwood, J. Neill, The influence of vibration on the creep of lead,** *Proc. Amer. Soc. Test. Mat.* **49,** 834–856, 1949.

Paper shows that vibration increases rate of creep and also accelerates recrystallization under stress. During recrystallization, the creep rate increases considerably. It is shown that both silver and copper reduces susceptibility of lead to recrystallize under stress, at least up to 15% extension. Test data are given to show that annealing lead at 120 C for 24 hr stabilizes crystal grains in some way, reduces creep rate under given conditions, and reduces susceptibility to recrystallization under stress.

Joseph Marin, USA

259. **Graf, O., Tests of the strength of welded sheets of aluminum alloys in tension and under repeated tensile loading** (in German), *Schweissen Schneiden* **1,** 11, 183–189, Nov. 1949.

Effect is investigated of butt welds, manufactured by either gas welding or arc welding, on tensile strength and tensile fatigue strength of certain structural aluminum alloys (Al-Mg-Mn and Al-Mg-Si). Test results obtained on specimens cut from 15-mm plates show considerable reduction of both ultimate tensile strength (36 to 70% reduction) and tensile fatigue strength for 2 million load repetitions (55 to 80% reduction). Strength of gas-welded joints is, in general, superior to that of arc-welded joints.

A. M. Freudenthal, USA

260. **Freeman, J. W., Reynolds, E. E., Frey, D. N., and White, A. E., The influence of conditions of heat treatment and hot-cold work on the properties of low-carbon N-155 alloy at room temperature and 1200 F.,** *Proc. Amer. Soc. Test. Mat.* **49,** 618–646, 1949.

See Rev **3,** 74.

261. **Parker, Earl R., High temperature properties of the refractory metals,** *Trans. Amer. Soc. Metals* **42,** 399–404, 1950.

This is a partial survey of what is known about mechanical properties of refractory metals. One table shows melting point, density, modulus, tensile strength, yield strength, elongation, and reduction-in-area at room temperature for these metals, together with properties of iron and nickel for comparison. One figure shows the variation in tensile strength and percentage elongation as functions of temperature. E. Saibel, USA

262. **Teed, P. Litherland, The properties of metallic materials at low temperatures,** New York, John Wiley & Sons, Inc., 1950, vii + 222 pp. $3.50.

Following a brief introduction and a short chapter on the structure of metals, author gives an excellent compilation of experimental results of investigations of low temperatures upon the principal mechanical properties of materials—proportional limit; yield and ultimate stress in tension, compression, and shear; elongation and reduction in area; notched bar impact; notch sensitivity; fatigue properties of notched and unnotched specimens; modulus of elasticity; shear modulus, bulk modulus, and Poisson's ratio; electrical conductivity; and, finally, the behavior of welds. Specific materials discussed are aluminum and its alloys, carbon and alloy steels, magnesium alloys, copper, nickel, tin, zinc, lead and their alloys. References are numerous and quite complete. Data are shown mostly in form of tables.

E. Saibel, USA

Mechanics of Forming and Cutting

(See also Rev. 240)

263. **Smith, Christopher, The extrusion of aluminium alloys,** *J. Inst. Metals* **76,** 429–451, 1950.

Clear and useful survey of metallurgical problems encountered in extrusion. Nice streamline photographs.

C. Zwikker, Netherlands

264. **Drucker, D. C., An analysis of the mechanics of metal cutting,** *J. appl. Phys.* **20,** 1013–1021, Nov. 1949.

The two-dimensional cutting process is qualitatively discussed mainly with regard to what happens to metal as it crosses the "shear plane" which extends from tool point diagonally upward to free surface. It is pointed out that metal is strained at a very high rate in cutting process, and, further, that nearly all of the

strain occurs in vicinity of shear plane. Small change in shear stress accompanying large shear strain which the metal undergoes as it crosses shear plane is regarded as a paradox. Author assumes that this can only mean that metal behaves as an ideal plastic.

State of stress on shear plane is examined and it is concluded that normal and tangential components of stress are constant for all points across shear plane. However, Mohr's circle of stress shows that shear plane cannot be a plane of maximum shear stress, as it undoubtedly is, unless normal stresses σ_x and σ_y are equal. This calls for zero normal stresses at free surface end of shear plane and maximum normal stresses at tool end.

Effect of cutting speed, rake angle, and depth of cut upon continuity of chip is discussed from point of view that a maximum shearing strain exists for continuity, and that this maximum allowable strain increases with rate of strain and pressure of normal stress on shear plane (i.e., hydrostatic pressure).

Experimentally observed fact that resultant cutting force per unit area of metal cut increases as depth of cut is decreased, is assumed to be due to inhomogeneity of the metal cut. It is shown that a great density of planes of weakness results for large depths of cut rather than for small depths, and it is inferred that metal will shear more readily when density of planes is large. While a reason for increased ease of shearing with increased density of shear planes is not given, strain hardening might be suggested. However, this possibility is excluded by author's previous conclusion that metal behaves as an ideal plastic.

Article is concluded by a brief discussion of depth of plastic flow in work piece and energy considerations in cutting.

Milton C. Shaw, USA

Hydraulics; Cavitation; Transport

(See also Rev. 401)

265. Jorissen, L., Pressure losses in new commercial steel pipes, *Int. Assn. for hyd. Str. Res.* **III-1,** 13 pp., Sept. 1949.

Account of an experimental research carried out in Liége to find what relation, if any, exists between Nikuradse's artificial roughness and industrial roughness. Average height of irregularities has been measured by means of a pneumatic feeler and correlated with pressure loss measurements on industrial new steel pipes of 60, 80, 100, and 150 mm nominal diameter.

The experiments show that in the quadratic zone the value of $\lambda(=4r_0g\Delta h/LV^2)$ is given with good accuracy by Nikuradse's formula, and in the transition zone the law for industrial pipes is intermediary between Nikuradse's law of homogeneous roughness and Colebrook and Withe's law of heterogeneous roughness.

Giulio De Marchi, Italy

266. Remenieras, G., Predetermination of energy loss in a water pressure tunnel from the loss at air flow in the same tunnel (in French), *Int. Assn. for hyd. Str. Res.* **III-2,** 22 pp., Sept. 1949.

On account of relatively high costs in lining rock tunnels, interest in unlined tunnels has increased in France and elsewhere. Author proposes to solve difficulties in predicting roughness for dimensioning by arranging an air-flow experiment in part of the tunnel, conducted so early that it is still possible to alter section and roughness if necessary. Author discusses earlier work on roughness of rock walls: Strickler-Manning coefficient 26–30 (corresponding to equivalent sand roughness 1.0–0.4 m, in conformity with recent Swedish material known by reviewer). Further, theoretical basis is given for experiments: dependence of resistance number on Reynolds number R_e and relative roughness, necessary R_e to ensure full turbulence, and choice of site of measuring length (see following review). Finally, paper gives a table of Strickler-Manning coefficients for various materials, some values of equivalent sand roughness after Moody, and an instruction for dimensioning, supplemented with diagrams.

H. T. Kristensen, Denmark

267. Bourguignon, P., Predetermination of energy loss in a water pressure tunnel from the loss of air flow in the same tunnel. Check of the validity of the method on the tunnel R.G. of Pont-Escoffier hydro-electric power plant (in French), *Int. Assn. hyd. Str. Res.* **III-9,** 18 pp., Sept. 1949.

To gain experience in method proposed by Remenieras (see preceding review), flow experiments have been carried out with air and with water in a length of the unlined rock tunnel mentioned in title. Author, who conducted the experiments, describes the arrangement and methods of measurement and gives results. Difficulties are discussed in detail (e.g., see page of water during air flow and air enclosed in the anfractuosities of ceiling during water flow). Author concludes that air experiments have determined resistance number with an exactness of ±7 or 8%. According to reviewer's calculation, results correspond to an equivalent sand roughness of 0.5 or 0.6 m in tunnel, or a Strickler-Manning coefficient of 29. Nothing is said concerning nature of rock and method of blasting.

H. T. Kristensen, Denmark

268. Silber, R., On the flow in canals (in French), *C. R. Acad. Sci. Paris* **229,** 1124–1126, Nov. 1949.

A step method is briefly explained for backwater curves in uniform and nonuniform channels.

Giulio De Marchi, Italy

269. Roy, S. K., Note on the stream depth at the edge of a free overfall (in French), *Houille blanche* **4,** 832–837, Nov.–Dec. 1949.

Let B be the point where lower free surface leaves bottom, and E point of upper free surface located vertically above B. A first approximation to the two-dimensional field is obtained by consideration of: (a) The singularity at B; this is of nature of a free streamline with constant velocity; (b) relations (due to force of gravity) between velocities at B and E and discharge passing between these points. Author deduces relative height BE and the velocity distribution along BE: he compares this solution with that obtained by Southwell and Vaisey using relaxation method.

A. Craya, France

270. Poggi, Bruno, On step waste-weir dischargers (in Italian), *Energia Elett.* **26,** 600–604, Oct. 1949.

Author studies case of a step waste-weir discharger formed by a cascade of spilling dams (Creager profiles and buckets for energy destruction). He calculates gradient to be given to line joining two successive spilling dams in function of discharge per unit of weir width. He uses Bernoulli equation, equation of momentum (hydraulic jump), and formula by Smetana giving length of stable jump.

Charles Jaeger, England

271. Cronin, J. D., and Hansen, W., An experimental study of the hydraulic downpull on a coaster gate, *J. Instn. Engrs. Austral.* **21,** 125–132, July–Aug. 1949.

One of the main results of author's small scale study is the measurement of the reduction on downpull obtained by providing holes in bottom girder of gate; more effective holes in this respect are those in center of beam.

A. Craya, France

272. U.S. Army, Corps of Engineers, **Wave action and breakwater location Oswego Harbor, New York, model investigation,** *Wways. exp. sta. tech. memo* 2-291, v + 19 pp., 2 tables, 24 plates, July 1949.

273. **Schaafsma, N. D. R., Settling as part of water purification** (in Dutch), *Ingenieur* **62**, 4, G 1-5, Jan. 1950.

Author proposes the formula $V_s = V_w(1 - C)$, where V_s is the velocity relative to a fixed datum and V_w is velocity relative to the water level of hydrated particles, concentration C, in the settling-tank process of water purification. Basic assumption is that particles may be treated as equal spheres which do not interfere. Experiments by author confirm this formula for concentrations lower than 0.71 and show, further, that for the absolute concentration there may be substituted the ratio of thickness of layer of compressed sludge, after three days settling, to height of original suspension column, multiplied by a constant. This constant has the value 4 for the precipitate obtained by introducing alum at *pH*-value 6.4. For concentrations higher than 0.71 the formula fails, since particles then interfere and, ultimately, insufficient water is present to insure complete hydration.

L. M. Milne-Thomson, England

274. **Parthilla, Ernst, Sedimentation tanks with constant velocity of flow** (in German), *Z. Öst. Ingen.-Archit.-Ver.* **94**, 166-169, Nov. 1949.

In sedimentation tanks a practically constant velocity of flow must be maintained for large variability of discharge. Paper investigates shape to be given to cross section of tank for this condition to be satisfied, for some types of sections of downstream channel. Some numerical examples are given.

Duilio Citrini, Italy

275. **Meyer-Peter, E., A few problems concerning the transport of solid matter in alpine and subalpine rivers** (in French), *Houille blanche*, no. spec. B, 688-706, 1949.

Article reviews briefly the present status of problem of bed-load transportation in experimental channels and applies results to alpine and subalpine rivers, particularly the Rhone. Meyer-Peter's contributions are well known and present article continues his earlier studies and includes results of new experiments in model channels. Several equations are developed including one concerned with Strickler's rugosity coefficient. Most of experiments deal with uniform sands. Du Boy's formula of bed-load traction is stated to be unsatisfactory, although it is much used in this country. Article closes with a set of results from 14 experiments on erosion produced by flow through a dam opening and gives an empirical formula for measuring degree of erosion produced. This should be of much value. Article contains many illustrations, though unfortunately the half-tone figures showing erosion and deposition of alluvia leave much to be desired.

J. M. DallaValle, USA

276. **Wuest, W., Box spring pressure gage** (in German), *Arch. tech. Messen*, no. 164, p. 71, Sept. 1949.

Article includes: Limits of use; accuracy of measurement (±1 to 2%); construction of the gage; transmission of the readings: mechanical, recording, and electrical systems; gages for measurement of absolute pressures, for differential pressures; and temperature compensation devices.

A. Schlag, Belgium

Incompressible Flow: Laminar; Viscous

(See also Revs. 273, 375, 469)

277. **Belyakova, V. K., On the stability of flow of a viscous fluid in a straight round pipe** (in Russian), *Prikl. Mat. Mekh.* **14**, 105-110, Jan.-Feb. 1950.

Author presents solution of the nonlinear differential equation for viscous flow in round tube. The linearization is obtained by method of small perturbations. A new set of boundary conditions (different from those used by Th. Sexl, Germany, 1927) is introduced and the stability parameters are evaluated.

A. Petroff, USA

278. **Viguier, G., Some remarks on a problem of J. Boussinesq** (in French), *Acad. roy. Belgique. Bull. Cl. Sci.* (5) **36**, 71-76, 1950.

Author studies one-dimensional motion of a viscous incompressible fluid in the case where velocity gradient is so large that the classical linear equation can no longer be used. If motion starts from rest initially over a plane in presence of a constant body force, the no-slip condition cannot be satisfied. Nevertheless, such a mathematical problem is considered and the differential equation is shown to be reducible to that of Riccati.

Courtesy of Mathematical Reviews Y. H. Kuo, USA

279. **Gerber, R., On the reduction to a variational principle of the equations of motion of a viscous incompressible fluid** (in French), *Ann. Inst. Fourier Grenoble* **1** (1949), 157-162, 1950.

Noting that previous attempts to obtain a variational principle equivalent to Navier's dynamical equations for viscous incompressible fluids have followed the Eulerian plan, author proposes instead to let volume of fluid under consideration be a material volume. For the rather special class of functions to be varied which he considers, he obtains a negative result.

C. Truesdell, USA

280. **Berker, Ratip, Inequality verified by the kinetic energy of a viscous incompressible fluid in a limited spacial domain** (in French), *Bull. Tech. Univ. Istanbul* **2**, 41-50, 1949.

A detailed version of a paper which appeared previously [*C. R. Acad. Sci. Paris* **228**, 1327-1329, 1949; Rev **3**, 511].

Courtesy of Mathematical Reviews Y. H. Kuo, USA

281. **Borodin, V. A., and Dityakin, Yu. F., Unstable capillary waves on the surface of separation of two viscous fluids** (in Russian), *Prikl. Mat. Mekh.* **13**, 267-276, 1949.

282. **Konakov, P. K., Some laws of the turbulent motion of liquids in pipes** (in Russian), *Izvestiya*, no. 10, 1456-1468, Oct. 1949.

By methods essentially the same as those presented in Goldstein's "Modern developments in fluid dynamics," **11**, pages 340-344, author derives formula given there for velocity defect for turbulent flow through a circular pipe. He also gives an equation for the friction coefficient, $1/\zeta^{1/2} = c_1 \ln Re + c_2$, which seems to fit Nikuradse's data well. However, derivation as carried out actually implies the usual law, $1/\zeta^{1/2} = c_1 \ln \zeta^{1/2} Re + c_2$, and not the one above. A considerable amount of manipulation of equations of motion seems to reviewer to obscure rather than clarify fundamental assumptions involved.

J. V. Wehausen, USA

283. Hahnemann, H. W., Resistance to the flow of a fluid through pipes and duct elements. A comprehensive report on some recent publications (in German), *Forsch. Beb. Ingenieurwesens* (B), **16**, 4, 109–112, 1949–1950.

284. Miles, John W., Quasi-stationary thin airfoil theory. *J. aero. Sci.* **16**, p. 440, 1949.

Author writes approximate formulas for lift and moment coefficients of a two-dimensional thin airfoil performing steady oscillations in an incompressible fluid. These are obtained from well-known results of unsteady airfoil theory but only first-order (in reduced frequency k) terms are retained. For plunging motion (oscillations in vertical translation), the so-called quasisteady formulas, which are taken from steady airfoil theory and simply neglect instationary effects, are found to be correct to order $k^2 \log k$. For pitching motions about the quarter chord, however, errors of order $k \log k$ and k are committed when the quasisteady formulas are used. Note ends with a discussion of practical significance of this, and a comment on compressibility corrections.

W. R. Sears, USA

285. Berman, Ya. R., The separation of streamlines from a circular cylinder in a bounded stream (in Russian), *Prikl. Mat. Mekh.* **13**, 543–546, 1949.

Author extends method used by Brodetsky [*Proc. roy. Soc. Lond. Ser. A* **102**, 542–553, 1923] for approximating the discontinuous flow about a circular cylinder in an infinite fluid to case where cylinder is situated midway between two parallel walls. A graph is given for the drag coefficient and angle of separation as functions of ratio of cylinder diameter to canal width. When the formulas are interpreted in terms of cavitation number, they seem to show good agreement with experiment.

J. V. Wehausen, USA

286. Scholz, N., Contribution to the theory of the lifting surface (in German), *Ingen.-Arch.* **18**, 2, 84–105, 1950.

Consider a two-dimensional flat-plate airfoil of chord l at incidence in a stream. If circulation is substituted by a single vortex at the forward quarter-point ($l/4$), and the streaming condition is satisfied at the rear quarter-point ($3l/4$), total lift and moment are unaltered. If plate is divided into n equal strips (l/n) and the corresponding substitution is made for each strip, same result holds for $n = 1,2,3,4$, and presumably for $n > 4$, although no proof is offered of this case. This strip method applied to an airfoil of finite aspect-ratio yields an integro-differential equation. This is replaced by nm linear equations by satisfying the streaming condition at m points ($3l/4n$) of each strip. The coefficients depend upon the downwash angles at these points. By assuming continuous chordwise distribution of circulation these downwash angles can be calculated once for all, and tables are given of their values corresponding to first three terms of a Fourier series. Detailed results are also shown for circular arc and S-profiles when plan form is rectangular.

L. M. Milne-Thomson, England

287. Campbell, W. E., Two electrical analogies for the pressure distribution on a lifting surface, *Nat. Res. Counc. Canada Mech. Engng.* Rep. MA-219, 45 pp., Oct. 1949.

In this theoretical report, determination of lift distribution for a given shape and attitude of a lifting surface is treated either as a boundary-value problem for disturbance velocity potential Φ, or for acceleration potential φ, which both satisfy Laplace's equation. On the surface $\partial\Phi/\partial z$ is equal to slope of surface, and in wake Φ should be independent of x and equal to value at trailing edge in same line parallel to x-axis. Outside the surface and wake $\Phi = 0$.

The derivative $\partial\varphi/\partial z$ is equal to curvature of surface, and φ is zero outside lifting plane and at trailing edge. If φ or Φ is replaced by an electric potential V, electric current being given by $\vec{j} = -\sigma \operatorname{grad} V$, these conditions can be interpreted as boundary conditions for either V or $\vec{j}$. By suitable configurations of electrodes, simulating surface and wake in first case, and (writing the boundary condition for φ on surface as a prescribed value of $\int_{-\infty}^{x} \partial\varphi/\partial z\,dx$) the fore region and surface in second case, the electric problem is solved by suitable adjustment of boundary potentials. Singularities at leading edge or at hinge axis of a control surface are avoided by adjustment of suitably shaped electrodes, form of which is determined by calculating mathematically the form of equipotential lines in neighborhood of such a singularity. Similarly, shape of electrodes at tips of finite wings is determined. Finally, effects of wall boundaries are examined.

R. Timman, Holland

288. Bilharz, Herbert, On the theory of the lifting line with periodic circulation (in German), *Z. angew. Math. Mech.* **29**, 311–317, Oct. 1949.

Prandtl's integrodifferential equation for vorticity along a lifting line is solved for a periodic distribution of downwash on the line. The complete downwash field is calculated and expressed in modified Bessel functions of second kind in two particular cases (simple cosine distribution, and piecewise constant downwash on the line). Graphs are given for vorticity distribution, induced resistance, and downwash velocity in a plane of symmetry.

R. Timman, Holland

289. Bilharz, Herbert, On the application of Dirichlet's discontinuity factor in flow theory (in German), *Arch. Math.* **2**, 27–32, 1949.

The integral equation of Prandtl's lifting-line theory is solved in two special cases. In the first problem, the vorticity distribution along the y-axis is found which makes the angle of attack the following step function: $\alpha(y) = 2\delta$ for $0 \leq |y| < l$, $\alpha(y) = \delta$ for $|y| = l$, $\alpha(y) = 0$ for $|y| > l$. The integral equation is solved by Fourier analysis. In the second problem, α is also a simple step function which is odd in y. Graphs of the vorticity for several values of the parameters of the step function are presented. It should be noted that the wings considered are three-dimensional but of infinite span. The angle of attack varies spanwise (i.e., in the y-direction) but is prescribed for all values of y.

Courtesy of Mathematical Reviews P. A. Lagerstrom, USA

290. Gurney, C., Surface forces in liquids and solids, *Proc. Phys. Soc. Lond. Sect. A*, **62**, 639–648, Oct. 1949.

Author's recent treatment of forces in surfaces of liquids is extended to surfaces of solids and to interfaces between solids and liquids. Solid surfaces formed by cleavage at temperatures such that no appreciable migration of molecules takes place, will usually be under a state of uniform stress, not necessarily tensile and not numerically equal to free energy of surface. If appreciable atomic migration takes place, surface of a crystalline solid melts, and solid is covered with a thin liquid film under a tension force greater than that of corresponding supercooled liquid, and such that chemical potential of the molecules in liquid film is equal to that of molecules in the crystal. This tension force is numerically equal to free energy of surface. If such a solid is subsequently cooled to a temperature at which atomic migration effectively ceases, it will have frozen in its surface a tension force corresponding to thermal equilibrium at some higher temperature. The Laplace Q force is discussed in terms of modern theories of

atomic bonding and applied to case of equilibrium at angle of contact between solid and liquid. From author's summary

291. **John, Fritz, The action of floating bodies on ocean waves,** *Ann. New York Acad. Sci.* **51,** 351–359, 1949.

This is an expository lecture describing the results in the paper of Rev 2, 1521, as well as some of those that presumably will be in part II of that paper, as yet unpublished. These additional results treat the case of motions in a liquid of infinite depth which are periodic and simple harmonic functions of the time. From the brief description there appears to be some duplication of work by Haskind [*Prikl. Mat. Mekh.* **10,** 33–66, 1946].

J. V. Wehausen, USA

292. **von Kármán, Theodore, Accelerated flow of an incompressible fluid with wake formation,** *Ann. mat. pura Appl.* (4) **29,** 247–249, 1949.

Author determines an example of a symmetrical unsteady flow with finite closed wake (of constant pressure) behind a flat plate. The flow problem involved differs from the known steady free boundary problem in that the nonstationary form of Bernoulli's equation replaces condition of constant flow speed as boundary condition on free streamlines. In author's example, complex velocity potential $F(z)$ is of the form, $F(z) = U(t)f(z)$, ($U(t)$ = velocity at ∞); flow therefore has property that wake is of constant shape. The flow is given in parametric form by

$$F = UAh[1 + \tfrac{1}{2}(\zeta + (1/\zeta))]$$

$$\frac{dF}{dz} = U(1+\zeta)[(1-\zeta/k)/(1-k\zeta)]^{1/2}, \quad |\zeta| \leq 1, Im\zeta \geq 0$$

where A and k are known numerical constants, and h is half width of plate. In order to satisfy the conditions on flow problem $U(t)$ must be of special form $U(t) = U(0)/[1 - (U(0)t/hkA)]$.

[Reviewer's note: It is possible to derive the entire set of flows with finite closed wake, and in the process to show that above example is one of a class of flows having a wake of constant shape; this is contrary to author's remark that his example is probably the only flow with this property.]

Courtesy of Mathematical Reviews D. Gilbarg, USA

293. **García, Joaquín Ocón, The systems of units in fluid dynamics** (in Spanish), *Rev. Cienc. apli.* **3,** 11, 430–435, Nov.–Dec. 1949.

Some considerations are given to different systems of measures. To avoid errors, a mechanical system is proposed, with four independent units of length, time, force, and mass. Reviewer believes this system, called technical by author, is of doubtful logical construction. Some dimensional inexactitude in English treatises of chemical engineering are noted.

Duilio Citrini, Italy

294. **Kirkwood, John G., Buff, Frank P., and Green, Melvin S., The statistical mechanical theory of transport processes. III. The coefficients of shear and bulk viscosity of liquids,** *J. chem. Phys.* **17,** 988–994, Oct. 1949.

A molecular theory of the coefficients of shear and bulk viscosity of monatomic liquids is developed on basis of the general theory of transport processes presented in first article of this series [Kirkwood, John G., title source, **14,** 180–201 (1946)]. With use of the Lennard-Jones potential and a reasonable analytic approximation to experimental radial distribution function, theory leads explicitly to ratios of coefficients to friction constant of theory of Brownian motion. With a preliminary estimate of the friction constant, a value of 1.27×10^{-3} poise for shear viscosity of liquid argon at 89°K has been calculated, in fair agreement with the experimental value, 2.39×10^{-3} poise.

Serge Gratch, USA

295. **Rubinstein, L. I., On a hydrodynamic problem** (in Russian), *Doklady Akad. Nauk SSSR* **68,** 5, 821–824, Oct. 1949.

The hydrodynamic problem is that of motion of water in a long rectangular canal caused by a wind blowing perpendicularly to canal and exerting only a frictional force on the free surface. Motion is taken as steady and two-dimensional, inertial forces are neglected in Navier-Stokes equations, and free surface elevation and slope are assumed small. With canal parallel to axis of earth, Coriolis force is introduced. This problem, in an oceanographic context, has been considered previously by Arakawa and Leibenson but without exact boundary conditions on side walls or Coriolis force. Author's method leads to a trigonometric series for stream function. Determination of coefficients requires solution of an infinite set of linear equations, and it is shown that the equations have a unique solution.

J. V. Wehausen, USA

296. **Lomer, W. M., The forces between floating bubbles and a quantitative study of the Bragg "bubble model" of a crystal,** *Proc. Cambr. philos. Soc.* **45,** 660–673, Oct. 1949.

Bubble rafts, when under sufficient stress, are known to exhibit behavior which is qualitatively similar to that of plastic deformation of single crystals. Present paper has as its aim the development of a quantitative theory of behavior of such rafts when subjected to small stresses. Developed theory is primarily an application of laws of surface tension to this situation and is based in part, according to author, on as yet unpublished work of M. M. Nicolson. Validity of theory is limited to small bubbles where slope of bubble cap is small. It is shown that raft behaves as an isotropic body; expressions for Young's modulus and Poisson's ratio are derived and are computed for various values of $R(\rho g/T)^{1/2}$, where R is a nominal bubble radius and T is surface tension. Experiments performed by author provide satisfactory agreement with theory within its limitations both for certain geometrical properties of bubbles and for Young's modulus and Poisson's ratio.

J. V. Wehausen, USA

297. **Katzoff, S., Finn, Robert S., and Laurence, James C., Interference method for obtaining the potential flow past an arbitrary cascade of airfoils,** *Nat. adv. Comm. Aero. Rep.* 879, 18 pp., 1947.

A rapidly converging iterative method is developed for calculating two-dimensional potential flow of an incompressible fluid through a cascade of arbitrary airfoils. Flow through the cascade is decomposed into four component flows: (1) Flow over one of the airfoils, as an isolated airfoil in the vector mean cascade flow. (2) A "disturbance" flow due to presence of all other airfoils in cascade. (3) A "compensating" flow which makes airfoil a streamline in flow made up of first two components. (4) A circulating flow about the airfoil which satisfies the Kutta condition at trailing edge.

The isolated airfoil flow is computed by any of existing methods. Disturbance flow component is synthesized by a finite number of sources, sinks, and vortexes placed within the contours of the other airfoils. Computations are simplified by a universal chart of flow field of an infinite row of logarithmic singularities with the central one missing. An alternate and more exact method represents the airfoils by a vortex distribution on airfoil surfaces. It is found that the more exact method may be useful in last step of iteration, but is unnecessary in preliminary steps. Compensating flow is readily computed by potential theory methods since its

stream function is the negative of the stream function of disturbance flow. Circulatory flow is computed such that stagnation point is at trailing edge in flow made up of isolated airfoil flow and compensating flow. Disturbance flow has only a small effect on conditions at trailing edge due to near symmetry.

Detailed consideration is given to methods of iteration, and to choice of location and strength of the disturbance-flow singularities.

Two examples are worked of compressor cascades. A method is given for calculation of flow at other inlet flow angles if flow is known at one inlet angle. W. G. Cornell, USA

298. Brinkman, H. C., Heat effects in capillary flow, I, *Appl. sci. Res.* **A 2,** no. 2, 120–124, 1950.

High rate of shear at a boundary in a viscous flow causes high energy dissipation. This produces a temperature rise and may explain variation of viscosity as measured by flow in a capillary.

Equating rate of generation of heat by friction to rate of dissipation of heat by conduction and convection in a Poiseuille flow gives a partial differential equation for which approximate solutions are given for the cases: (a) Walls at constant temperature, and (b) walls impervious to heat. Distribution of temperature is shown for each case as a family of dimensionless curves of temperature against radial distance for various axial distances along capillary. L. Landweber, USA

299. Munk, Walter H., The solitary wave theory and its application to surf problems, *Ann. New York Acad. Sci.* **51,** 376–424 1949.

Basic idea of paper is that the motion of water near crest and the energy in each wave length of a wave of finite height advancing into shallow water of decreasing depth can be calculated from those of a solitary wave of same height and mean local depth. With additional assumption that wave length and energy are conserved in propagation, the change in wave profile and amplitude can be calculated. Author then defines the breaking point as point where wave crest of corresponding solitary wave becomes pointed, the extreme use of the solitary wave. From this he deduces a relation between ratio of height of breaking to initial wave height, and ratio of initial height to wave length ("initial steepness"). Depth of breaking is then 1.28 times breaking height. (In reviewer's opinion, formation of breakers is determined by local velocity distribution in wave, while the solitary wave can be only used as a model to calculate the integrated quantities such as wave height and energy. Formation of breakers is actually a case of nonlinear wave propagation in shallow water as proposed by Stoker [*Communications on appl. Math.* **1,** 1–87, 1948; Rev **2,** 1520]. From this point of view, waves, if not damped out by friction, will break even with constant depth. Therefore, no unique correlation between ratio of breaking height to initial height and initial steepness can be expected. This is substantiated by extremely wide scatter of test points if so plotted.) Paper contains, also, calculations on wave refraction, longshore current, rise in sea surface, and sand transport.

H. S. Tsien, USA

300. Høiland, Einar, Stability and instability waves in sliding layers with internal static stability, *Arch. Math. Naturvid.* **50,** 3, 71–94, 1949.

Let an incompressible fluid be subject to a gravitational force, be bounded by two horizontal planes, and have a density varying according to the barometric law $\rho = \rho_0 \exp(-\Gamma g_n)$. Author considers stability of two-dimensional motion occurring when velocity of fluid has a discontinuity at a horizontal plane but is constant on either side. Linearized equations for perturbed motion and harmonic perturbations are assumed. Paper is devoted chiefly to a general discussion of properties of equation relating the velocity of propagation of disturbance and its wave length. J. V. Wehausen, USA

Compressible Flow, Gas Dynamics

(See also Revs. 87, 344, 351, 374, 380, 393, 468)

301. Ward, G. N., The supersonic flow past a slender ducted body of revolution with an annular intake, *Aeronaut. Quart.* 1, 305–318, 1950.

It is assumed that the fluid that enters intake continues to move supersonically, at least for a short distance. Solution of the linearized small-disturbance potential equation is undertaken by Heaviside's operational methods; appropriate solutions for outside flow are, in operational form, modified Bessel functions. For small radii of body surface only leading terms are retained. Flow at zero incidence is considered first, and both pressure coefficient and drag are found. Next, the additional effects due to incidence are calculated, including the normal force, lift, moment, and additional drag. All these are for external flow. Normal-force or lift contribution of internal flow is computed from momentum considerations. W. R. Sears, USA

302. Oswatitch, K., The velocity distribution at local transonic ranges on flat airfoils (in German), *Z. angew. Math. Mech.* **30,** 1/2, 17–24, Jan.–Feb. 1950.

In order to study the theory of thin airfoils near transonic range, author simplifies the equations of two-dimensional steady irrotational compressible flow to case in which the vertical component of velocity is small and the horizontal component of velocity is near the speed of sound. He obtains the nonlinear system $U_x + V_y = UU_x$, $U_y - V_x = 0$, where U and V are proportional to the differences between the x and y components of velocity locally and at infinity. The functions $U = \phi_x$, $V = \phi_y$ satisfy the second equation and, for the first, give $\phi_{xx} + \phi_{yy} = UU_x$. Author then applies the solution of the Poisson equation in a straightforward manner to get a nonlinear integral equation for U. The integral equation is solved approximately and the theory is applied to symmetric flow about an airfoil whose section is bounded by two equal circular arcs. Function U is calculated in several different velocity ranges. Rather pronounced shocks appear in transonic range. Author states that the results agree with experiment.

Courtesy of Mathematical Reviews E. Pinney, USA

303. Kiebel, J. A., Exact solutions of equations of gas dynamics, *Nat. adv. Comm. Aero. tech. Memo.* 1260, 12 pp., June 1950.

See Rev **1,** 505.

304. Bleakney, W., Fletcher, C. H., and Weimer, D. K., The density field in Mach reflection of shock waves, *Phys. Rev.* **76,** 323–324, July 1949.

A typical interferogram of a three-shock configuration obtained in the Princeton shock tube by means of a Mach-Zehnder interferometer is exhibited and discussed briefly. This is one of many such interferograms being obtained by authors in an effort to provide experimental data which may lead to a satisfactory theory concerning this phenomenon. Discrepancy between theoretical prediction and experimental data of the physical quantities in neighborhood of a three-shock intersection is one of the major unsolved problems in present-day supersonic aerodynamics.

Authors' interferometric work is of a very high quality and indicates unusual mastery of optical techniques.

Harry Polachek, USA

305. **Pillow, A. F., The formation and growth of shock waves in the one-dimensional motion of a gas,** *Proc. Cambr. philos. Soc.* **45**, 558–586, Oct. 1949.

Author considers Riemann's classical problem of compressible flow of gas in a cylinder, with emphasis on the discontinuous solution. The shocks make it necessary to introduce the entropy, $S = c_v \log p\rho^{-\gamma}$, which is assumed, however, to be fixed for each particle, between shocks. System becomes $dS = 0$ on $dx/dt = u$, and $d(u \pm \int c d\rho/\rho) \pm (\int c d\rho/\rho)dS/2\gamma c_v = 0$ on $dx/dt = u \pm c$. Confining his attention to some special case, e.g., piston constantly accelerated from rest, author determines values of u and ρ from shock-free approximation $u - \int c d\rho/\rho = \text{const}$, but he nevertheless assigns a value of S to each particle by Rankine-Hugoniot discontinuity conditions. This gives S, u, and ρ in an x, t region and, by elimination, it determines S in terms of u and ρ. Now with two dependent variables once again, author estimates the particle motion behind shock. He also estimates position of shock wave using Riemann's method of inverting role of dependent and independent variable so that shock becomes a known boundary. Lastly, he shows that after piston achieves a constant terminal velocity, resulting shock interaction phenomenon produces same final shock as produced by an impulsive start.

Harvey Cohn, USA

306. **Thomas, T. Y., The distribution of singular shock directions,** *J. Math. Phys.* **28**, 153–172, Oct., 1949.

Author has previously shown that there exist relations between local curvature k of a shock wave and its derivatives, and the curvature K of the streamline (behind shock wave) and its derivatives which are always given in the form

$$\frac{d^n K}{d\eta^n} = G_n(M, \alpha)\,\frac{d^n k}{ds^n} + H_n,\ n = 0, 1, 2, 3 \ldots$$

M is, as usual, the Mach number in a uniform region in front of shock wave, α inclination of shock wave to horizontal at point under consideration, and H_n are polynomials in k and its derivatives or order less than n; η and s represent arc lengths along streamline and shockline, respectively. Discussion in paper is confined to the vertex of a pointed obstacle, with attached shock waves. A value of α for which any of the functions $G_n(M, \alpha) = 0$ is defined to be singular relative to M ($M > 1$), since there will, in general, exist no shock-wave solution for a given shape of obstacle.

Main theorem of paper states that singular shock angles α are dense in an interval $\beta(M) < \alpha < 90°$. In other words, for values of α in that interval there always exists a value n such that $G_n(M, \alpha) = 0$. $\beta(M)$ is that value of α for which m, the Mach number behind shock wave, is exactly equal to unity. Furthermore, for interval $\beta(M) < \alpha < 90°$, $m < 1$; i.e., flow is subsonic. Explicit expressions for functions $G_n(M, \alpha)$ are derived.

H. Polachek, USA

307. **Bleakney, Walker, and Taub, A. H., Interaction of shock waves,** *Rev. mod. Phys.* **21**, 584–605, Oct. 1949.

Paper concerns mainly the interaction of a plane oblique shock with a straight wall. Depending upon angle of incidence and "strength" of shock, one has either a regular reflection or a Mach reflection, latter configuration forming a lambda shock. Schlieren and interferometer pictures are shown for case of Mach reflection.

Hideo Yoshihara, USA

308. **Robbertse, W. P., and Burgers, J. M., Solutions of the equations for the nonuniform propagation of a very strong shock wave. I,** *Nederl. Akad. Wetensch. Proc.* **25**, 958–965, 1949.

A one-dimensional gas flow is characterized by $x = \varphi(s, t)$, where x and s are the Eulerian and Lagrangian coordinates respectively. A shock of very high intensity passes through the gas, with state of motion of the gas anterior to its passage supposed known. By applying Lagrangian labels in a special manner, a differential relation is found which holds along trace of shock. This relation permits the explicit solution of flow equations provided a particular form for this solution is assumed. In course of this solution only partial recourse is had to equation of motion. In order that this equation be fully satisfied it is necessary to postulate a specific density distribution in the unshocked gas. This distribution in turn dictates a shock velocity which increases with time.

Courtesy of Mathematical Reviews D. P. Ling, USA

309. **Robbertse, W. P., and Burgers, J. M., Solutions of the equations for the nonuniform propagation of a very strong shock wave. II,** *Nederl. Akad. Wetensch. Proc.* **52**, 1067–1074, 1949.

Continuing previous work [see preceding review], authors examine numerically the derived solution. The arbitrary constants appearing in this solution are related to physical processes involved and evaluated. Further, a numerical comparison is made between this solution and previous approximate solutions. Despite differences in boundary conditions among the various cases compared, only relatively minor differences appeared in early phases of process.

Courtesy of Mathematical Reviews D. P. Ling, USA

310. **Frankl, F. I., On the formation of shock waves in subsonic flows with local supersonic velocities,** *Nat. adv. Comm. Aero. tech. Memo* 1251, 8 pp., 1950. Translated from *Prikl. Mat. Mekh.* **11**, 199–202, 1947.

See Rev **3**, 949.

311. **Morley, A. W., The efficiency of compression in a single adiabatic shock,** *Aircr. Engng.* **21**, 320–321, 324, Oct. 1949.

Calculations of efficiency of compression in a single adiabatic shock wave are tabulated and plotted for ratio of specific heats equal to 1.40 and 1.33. Efficiency is defined as ratio of work done to compress the gas (from static conditions ahead to total head conditions downstream) to work done if process were carried out isentropically. For a given deflection of stream, efficiency of compression has a maximum at a particular upstream Mach number. Efficiency may be increased by dividing compression into a number of stages. For a given number of stages, maximum efficiency is reached when entropy increases of discrete steps are all equal.

C. C. Lin, USA

312. **Cabannes, Henri, Study of the singularity on the peak of an attached shock wave on a two-dimensional flow** (in French), *C. R. Acad. Sci. Paris* **229**, 923–925, 1949.

This is a note, including tantalizingly interesting equations and remarks, which has been rendered almost unintelligible owing to the compression demanded of authors according to regulations of the *Comptes Rendus*. Paper is concerned with heterentropic two-dimensional flow behind a stationary shock attached to an obstacle, neglecting viscosity and heat conduction. It seems likely to reviewer that paper, if expanded to about ten times its length (with figures, tables, graphs, etc.), would have added considerably to our knowledge, especially relating to flow

at large Mach numbers. But it is impossible to be certain of anything, even of this, when the complicated investigation is presented in two brief pages. M. J. Lighthill, England

313. Gurevich, M. I., Lift force of an arrow-shaped wing, *Nat. adv. Comm. Aero. tech. Memo.* 1245, 14 pp., Oct. 1949.

Solutions for supersonic pressure distribution and lift on triangular wings were published nearly simultaneously by Robinson, England; Stewart, USA; Brown, USA; and Gurevich, Russia. This report is a translation of Gurevich's paper. Analysis, based on Busemann's conical flow theory, is similar to that of Stewart. This study is somewhat broader in scope than some of others in that wings having supersonic, as well as subsonic, leading edges are considered. Treatment of drag is brief, however, being confined to a few remarks and to display of an integral for calculation of drag. John R. Spreiter, USA

314. Spreiter, John R., The aerodynamic forces on slender plane- and cruciform-wing and body combinations, *Nat. adv. Comm. Aero. Rep.* 962, 17 pp., 1950.

Load distribution, forces, and moments are calculated theoretically for inclined slender wing-body combinations consisting of a slender body of revolution and either a plane or cruciform arrangement of low-aspect-ratio pointed wings. Results are applicable at subsonic and transonic speeds, and at supersonic speeds, provided entire wing-body combination lies near center of Mach cone.

Analysis of slender cruciform-wing and body combinations results in following conclusions: Lift and pitching moment are independent of angle of yaw, and side force and yawing moment are independent of angle of attack. If vertical and horizontal wings are identical, rolling moment is zero for all angles of pitch and yaw. By symmetry considerations, these results are shown to be equally applicable for any cruciform-wing and body combination having identical horizontal and vertical wings of arbitrary plan-form and aspect ratio. From author's summary.

H. J. Allen, USA

315. Moskowitz, Barry, and Moeckel, W. E., First-order theory for unsteady motion of thin wings at supersonic speeds, *Nat. adv. Comm. Aero. tech. Note* 2034, 26 pp., 1950.

Determination of aerodynamic loading on thin wings executing unsteady motions in a supersonic stream is discussed, subject to the stated restriction that the second time derivative of the perturbation motion is negligible. Results are applied to pitching and plunging motions of a swept wing with straight, supersonic, parallel leading and trailing edges and streamwise tips. In opinion of reviewer, the method is identical with that previously known [J. W. Miles, *J. aero. Sci.* **16**, 378–379, 1949], although authors claim more generality. The essential restriction (in addition to those implied by linearization), albeit not explicitly stated either by authors or in cited reference, is that the trailing edge is nowhere subsonic, so that first-order frequency terms associated with wake exert no influence at the wing.

J. W. Miles, USA

316. Leray, Jean, Compressible fluids. Application of the Tchapliguine method to the wing of infinite span (in French), *J. Math. Pures appl.* (9) **28**, 181–191, 1949.

Author extends problem of H. S. Tsien to case of finite circulation by hodograph method with linear pressure-volume approximation. This same problem has previously been treated by C. C. Lin, L. Bers, A. Gelbart, P. Germain, and J. W. Craggs [see Craggs, "Ministry of Supply" [London], *Aeronaut. Res. Council Rep. and Memo.* 2273 (9801), 1949; Rev **3**, 312]. It is shown essentially that, in case of linear pressure-volume approximation, a two-dimensional irrotational flow of a compressible fluid about a body can be obtained by a point transformation from a known incompressible flow about a similar body with the same circulation and same condition at infinity. When the free-stream Mach number is less than 0.6 (this might be interpreted to mean well below the critical), this transformation reduces to that of Prandtl and Glauert.

Courtesy of Mathematical Reviews Y. H. Kuo, USA

317. Kraft, Hans, Reaction tests of turbine nozzles for subsonic velocities, *Trans. Amer. Soc. mech. Engrs.* **71**, 781–787, Oct. 1949.

Paper supplements one by J. H. Keenan entitled "Reaction tests of turbine nozzles for supersonic velocities" [title source, same issue]. Both papers deal with a series of tests of steam nozzles, made by the General Electric Company from 1920–1928, which were characterized chiefly by use of reactive thrust measurements as a means of determining nozzle performance. Tests are reported at this late date because of current interest in gas turbines and reactive phenomena, and because it is believed that the data are still valid and constitute a solid foundation for further research. In addition to performance data, presented chiefly in form of curves of velocity coefficient plotted against isentropic velocity (latter ranging up to 3500 fps), results are interpreted to some extent in terms of modern aerodynamic theory. Importance of boundary-layer phenomena is especially emphasized. A discussion is given of effects of nozzle partition, angle, pitch, aspect ratio, and edge thickness. It is recognized that data from such tests may be affected by "end" losses which would not occur in a conventional circular nozzle diaphragm, and that time effects introduced by rotating elements were absent, but it is stated that, whenever tests indicated a gain, a corresponding gain was obtained in performance of complete turbine, though not necessarily of same magnitude. C. W. Smith, USA

318. Reid, Elliott G., Annular-jet ejectors, *Nat. adv. Comm. Aero. tech. Note* 1949, 98 pp., Nov. 1949.

Paper presents an experimental investigation of entrainment and thrust augmentation characteristics of ejectors which incorporate annular nozzles. Tests results show that ejectors with annular nozzles and nondivergent mixing tubes have negligible thrust augmentation capabilities, despite possession of entrainment characteristics substantially identical with those of conventional ejectors with central nozzles. The augmentation deficiency is ascribed to the inequality of frictional forces experienced by the two types. Combination of divergent mixing tubes with annular nozzles was found to result in substantial improvement of both entrainment and augmentation characteristics. While augmentation so effected still falls short of that attainable with comparable central nozzles and straight mixing tubes, corresponding entrainment characteristics are superior to any thus far demonstrated by central nozzle types. This advantage is believed to originate in suppression of flow separation from walls of diffusers by scouring action of annular jets.

Report covers, substantially, test data, and included are results of total pressure surveys made at downstream end of mixing tubes. The appendix presents a mathematical treatise on correlation between thrust correction factor and primary nozzle discharge coefficient. Report represents a definite contribution to the understanding of physical characteristics of annular-jet ejectors. H. E. Sheets, USA

319. **Perry, J. A., Jr., Critical flow through sharp-edged orifices,** *Trans. Amer. Soc. mech. Engrs.* **71,** 757–764, Oct. 1949.

Experimental results are given for compressible flow through five sharp-edged orifices ranging from $^5/_{32}$ in. to $^1/_4$ in. diam, in a 2-in-diam pipe line. Flow coefficient is referred to an ideal jet, having area of orifice, expanded isentropically from upstream pressure and temperature to a final pressure equal either to actual back pressure or to critical pressure, whichever is higher. Tests are performed over a range of pressure ratios (back pressure to supply pressure) from 0.87 to 0.14.

Flow coefficient had customary value of about 0.60 for pressure ratios approaching unity, and appeared to approach value 0.84 as pressure ratio approached zero. At critical pressure ratio (0.53), flow coefficient was 0.74. Character of flow seemed different for pressure ratios less than and greater than critical, as evidenced by curve of flow coefficient vs. pressure ratio being concave downward for former and concave upward for latter.

Ascher H. Shapiro, USA

320. **Keenan, J. H., Reaction tests of turbine nozzles for supersonic velocities,** *Trans. Amer. Soc. mech. Engrs.* **71,** 773–780, Oct. 1949.

Paper presents tests of turbine nozzles by reaction method. Results of tests are compared with each other and with analytical values. In addition to usually assumed isentropic expansion from inlet state to exhaust pressure, velocities are also calculated by assuming that processes in nozzle are isentropic except for a normal shock which takes up a position in nozzles such as to cause stream to fill exit area at exhaust pressure whenever possible. Agreement of the test results with results calculated according to last method is quite good. Remaining disagreement which exists is attributed to separation at shock and to transmission of exhaust pressure effects upstream through boundary layer.

Test apparatus and validity of test results are discussed in detail. Paper is particularly useful because it presents considerable test data of great interest and of general value on nozzle cascades, relating to the flow through supersonic nozzles with both under and over expansion. Comments on this paper by various engineers and author's closure result in an interesting discussion.

H. E. Sheets, USA

321. **Falkovich, S. V., A class of deLaval nozzles,** *Nat. adv. Comm. Aero tech. Memo.* 1236, 13 pp., Oct. 1949.

A study is made of the irrotational adiabatic motion of a gas in transition from subsonic to supersonic velocities. A shape of De Laval nozzle is given, which transforms a homogeneous plane-parallel flow at large subsonic velocity into a supersonic flow without any shock waves beyond transition line from subsonic to supersonic regions of flow. Method of solution is based on expressing equations of motion in a certain form and integrating near transition line. Velocity potential and stream function can then be expressed in proper form so that they may be analytically continued into supersonic region of flow.

Nicholas Di Pinto, USA

322. **Bergman, Stefan, On two-dimensional supersonic flows,** *Nat. adv. Comm. Aero. tech. Note* 1875, 49 pp., 1949.

Author finds solution of the equations of plane steady irrotational supersonic gas flow, which involves an arbitrary function of a real variable.

M. J. Lighthill, England

323. **Sokolovskiĭ, V. V., On a transformation of the equations of plane flow of a gas** (in Russian), *Prikl. Mat. Mekh.* **13,** 253–256, 1949.

Author discusses a transformation of equations for plane irrotational flow. Subsonic and supersonic cases are discussed separately. Transformation is similar to those previously introduced by Chaplygin and others. Both physical plane and hodograph plane are considered. Author indicates how some boundary conditions may be expressed in terms of new coordinates. However, no specific problems are completely formulated. Neither does author indicate any special advantage of the new transformations for solving any boundary-value problem.

Courtesy of Mathematical Reviews P. A. Lagerstrom, USA

324. **Heaslet, Max A., and Lomax, Howard, Two-dimensional unsteady lift problems in supersonic flight,** *Nat. adv. Comm. Aero. Rep.* 945, 9 pp., 1949.

See Rev 2, 1529.

325. **Dorrance, W. H., Concerning linearized supersonic flow solutions for rotationally symmetric bodies,** *J. aero. Sci.* **16,** 508–509, 1949.

Author shows that form of velocity potential for linearized supersonic flow over rotationally symmetric bodies used by R. Sauer ["Theoretische Einführung in die Gasdynamik," Berlin, Springer, 1943; Ann Arbor, Mich., J. W. Edwards, 1945; translated as "Introduction to theoretical gas dynamics," Ann Arbor, Mich., J. W. Edwards, 1947; cf. pp. 72–81 of the translation] and C. C. Lin ["An introduction to the dynamics of compressible fluids," Tech. Rep. no. F-TR-1166ND (GDAM A-9-M I), Headquarters Air Materiel Command, Wright Field, Dayton, Ohio, 1947, in particular, pp. 89–90] is incorrect.

H. S. Tsien, USA

326. **Munk, M. M., The reversal theorem of linearized supersonic airfoil theory,** *J. appl. Phys.* **21,** 159–161, 1950.

Under certain conditions, as yet not completely clarified, a wing in supersonic flow has same drag and lift in original flow as in reversed flow according to linearized theory. Author gives a proof, based on simple symmetry arguments of this reversibility theorem. Class of wings for which this proof is valid is not too clearly defined. However, it is specifically assumed that pressure is computed from linearized Bernoulli's law so that no leading-edge suction is considered. For case of zero thickness (lifting case) this condition is satisfied if all edges are supersonic. In this case angle of attack has to be constant, and , in fact, it is very easy to give counterexamples showing that lift is changed by flow reversal if angle of attack is variable. Leading-edge suction is also zero when the average of local angle of attack on top and bottom surfaces is zero. Local lift is then zero, and the reversibility theorem holds in addition for the drag.

Courtesy of Mathematical Reviews P. A. Lagerstrom, USA

327. **Lighthill, M. J., The drag integral in the linearized theory of compressible flow,** *Quart. J. Math., Oxford Ser.* **20,** 121–123, 1949.

In many problems of compressible fluid flow a solution of the linearized equations may give a good approximation to the real flow except near certain singularities. In this paper a form of the momentum surface integral for drag is found which is correct however near to those singularities the surface is drawn. Idea of the proof is first to set up the integral over a surface which is sufficiently far away from singularities so that linearized equation may be considered valid. Then it is shown that, as a consequence of the equations for linearized potential, this integral is invariant under shrinking of surface. Similar considerations are applied to an integral expression for the side force.

Courtesy of Mathematical Reviews P. A. Lagerstrom, USA

328. Haskind, M. D., and Falkovich, S. V., Vibration of a wing of finite span in a supersonic flow, *Nat. adv. Comm. Aero. tech. Memo* 1257, 11 pp., 1950.

See Rev **1**, 353.

329. Miles, John W., On the oscillating rectangular airfoil at supersonic speeds, *J. aero. Sci.* **16**, p. 381, 1949.

This short note, presenting the results of an unpublished analysis, states that the pressure distribution on the tip of a rectangular wing carrying out arbitrarily distributed sinusoidal oscillations has been calculated in the linear-perturbation approximation. The final results are given here for the special case where the downwash is constant over a spanwise strip. This result differs from the two-dimensional result only by the appearance of an extra term. For vanishingly slow oscillations the known steady-flow formula is obtained. More detailed results are to be published elsewhere. W. R. Sears, USA

330. Miles, John W., Quasi-stationary airfoil theory in compressible flow, *J. aero. Sci.* **16**, p. 509, 1949.

Author uses Possio's integral equation [*Aerotecnica* **18**, 441–458, 1938] for the lift on a thin two-dimensional airfoil in periodic motion at subsonic speeds. Solutions correct to first order in the frequency are obtained for the technically important cases of pitching and plunging (vertical oscillations without pitching). It is found that, although the results reduce to steady-flow results at zero frequency, there are relatively large correction terms for small frequencies, and these are particularly important at high Mach number. A numerical tabulation shows that damping in pitch, for example, may be badly overestimated if steady-flow values are used. W. R. Sears, USA

331. Marshall, W. S. D., Levacic, I., Young, A. D., and Powter, G. J., Low-speed wind tunnel tests of a high-lift supersonic wing. *Aer. Res. Counc., Rep. Mem.*, R. & M. 2365, Sept. 1949, 24 pp., 1950.

Tests have been made on a rectangular wing of biconvex section 7.5% thick with a flap and leading-edge slat of rather unusual design. Flap was of slotted semi-Fowler type; its chord was equal to 0.25 of wing chord and it was set at an angle of 46 deg. Slat was formed from foremost 16% of section, which was moved forward from rest of section and rotated through an angle. A series of these slat positions and angles was tested in conjunction with flap set at 46 deg.

In view of the structural difficulties such a combination would involve, tests were also made using a simpler method for increasing the lift coefficient. This consisted in articulating the section at two hinges so as to form plain front and rear flaps of chords 0.164 and 0.254 of the wing chord respectively.

From authors' summary

332. Miles, John W., On nonsteady motion of delta wings, *J. aero. Sci.* **16**, 568–569, 1949.

The velocity potential $\phi(x, y, 0+)$ for an oscillating supersonic delta wing is written down according to Garrick and Rubinow [*Nat. adv. Comm. Aero. tech. Note* 1383, 1947; Rev **3**, 757]. It is shown that certain spanwise weighted integrals of ϕ can be reduced to integrals arising in two-dimensional problem. These are, in particular, the weighted integrals for lift and rolling moment on a spanwise strip. They are brought into the form of integrals involving Green's function of a plane flow. A similar device can be used when motion is not a steady oscillation but a motion beginning at an instant $t = 0$; in this case, Green's function is replaced by its Bromwich integral.

W. R. Sears, USA

333. Miles, John W., On harmonic motion at supersonic speeds, *J. aero. Sci.* **16**, 378–379, 1949.

Assuming harmonic time dependence, the linear-perturbation equation can be written $\psi_{yy} + \psi_{zz} = \beta^2\psi_{xx} + k^2\psi$, where the perturbation velocity potential is $\psi \exp[i(\omega t - \lambda x)]$, β^2 denotes $M^2 - 1$, M is the stream Mach number, ω angular frequency, $\lambda = Mk/\beta$, and $k = \omega/\beta c$. Here c denotes sound speed, t time, and x, y, z the usual coordinates, x being measured in stream direction. Author proposes to neglect $O(k^2)$ for slow oscillations; then steady-flow solutions can be employed. As an example, he calculates the moment on a flat two-dimensional airfoil executing pitching oscillations, using appropriate steady-flow Green's function, and obtains a result consistent with his previous work [e.g., title source, **14**, 351–358, 1947; Rev **1**, 156]. He also discusses the application of this approximate method to the rectangular wing tip and slender body of revolution.

W. R. Sears, USA

334. Thomas, T. Y., First approximation of pressure distribution on curved profiles at supersonic speeds, *Proc. nat. Acad. Sci. Wash.* **35**, 617–627, Nov. 1949.

Results are presented of pressure computations for a biconvex profile at supersonic speed. Method of calculation constitutes first approximation in a more general analytical procedure proposed by author. Employed equations are the same as would be obtained by assuming that, for a given free-stream Mach number, pressure at each point on profile is identical with that on a simple wedge tangent to profile at the point in question and traveling at same Mach number. Resulting pressure distributions agree closely with those given by more usual "shock-expansion" method. Walter Vincenti, USA

335. Sears, W. R., The linear-perturbation theory for rotational flow, *Rev. sci. Instrum.*, **21**, 268–271, Jan. 1950.

The perturbation stream function is considered for a nearly uniform field of compressible rotational flow, and an equation is derived for this function in terms of the vorticity. For subsonic flow, the familiar Prandtl-Glauert transformation can be readily applied, relating the rotational flow past a body to an incompressible flow past a thinner body. Equations relating vorticity, velocity, and body shapes in compressible and incompressible flows are derived. A. D. Young, England

336. Bleakney, Walker, Weimer, D. K., and Fletcher, C. H., The shock tube: a facility for investigations in fluid dynamics, *Rev. sci. Instrum.* **20**, 807–815, Nov. 1949.

Article gives the theory of operation of a shock tube of constant cross section and describes in detail the construction, instrumentation, and performance of a shock tube 4 by 18 in. in cross section and 38 ft long, at Princeton University. Interferograms of typical shock-tube phenomena are presented. One of most interesting pictures illustrates a new method developed for identifying fringes across the discontinuous shock front. In this method, light source is displaced so that light is oblique to shock by one or two degrees. Shock appears then as a band of finite width through which fringes are easily traced.

John V. Becker, USA

337. Prim, R. C., A new superposition principle for steady gas flows, *Quart. appl. Math.* **7**, 4, 445–450, Jan. 1950.

For inviscid, nonconducting gases, for which density is a function of pressure times a function of entropy, a formula is derived for obtaining steady spatial fields of flow from plane fields. This is based on so-called superposition and substitution principles

derived in earlier papers. Method is used to obtain a one-parameter family of generalized Beltramic fields ($W \times \text{curl } W = 0$) from any given plane flow. A. D. Young, England

338. **Teofilato, Stefano, Contribution to the study of a hydrogas dynamic similitude** (in Italian), *Monogr. Scient. Aeron.* no. 4, 11 pp., Jan. 1949.

339. **Truesdell, C. A., Bernoulli's theorem for viscous compressible fluids,** *Nav. Res. Lab. Rep.* 3558, 3 pp., Oct. 1949.

By a Bernoulli theorem is meant a statement that a certain finite expression, one of whose terms is specific kinetic energy $V^2/2$, a second of which contains one or more dynamical variables, and none of which involves a line integral, remains constant or a function of time only upon each member of a certain family of curves. In any motion of a viscous compressible fluid in which vorticity is steady, such a finite expression is given explicitly, and a construction for curves along which it is a function of time only is indicated. In special case of an incompressible fluid subject to conservative extraneous force, the finite expression reduces to that of classical Bernoulli theorem, and curves are defined in terms of velocity field only, so that, just as in classical hydrodynamics, Bernoulli theorem yields a formula for pressure in terms of purely kinematic observables. From author's summary

340. **Ginzel, J., The Pohlhausen method for the calculation of laminar compressible boundary layers on a heated wall** (in German), *Z. angew. Math. Mech.* **29**, 321–337, 1949.

Author extends the Pohlhausen method to boundary layer of a compressible fluid by including the energy equation. Polynomials of fourth degree are used to represent velocity and temperature distributions, with different values of boundary-layer thickness. Required calculation is thus integration of a system of two ordinary differential equations. Numerical examples worked out by this method show that skin friction is little influenced by heating and depends very little on law of viscosity. Heat transfer depends very much on heating. Process can be easily carried over to case of boundary layer over axially symmetric bodies. C. C. Lin, USA

341. **Gilbarg, David, A characterization of non-isentropic irrotational flows,** *Amer. J. Math.* **71**, 687–700, 1949.

Paper is unusual in that it attacks a problem of gas dynamics in the large. Objective is to prove that with certain specific exceptions any steady irrotational flow of an inviscid nonpiezotropic gas [$p = f(\rho, \eta), f_\rho \neq 0, f_\eta \neq 0$] which does not conduct heat is a flow of uniform specific entropy η. This result is often assumed in engineering literature on basis of intuitive arguments. From energy equation it follows at once that specific entropy of each particle remains constant throughout each region of continuous flow. Appropriate local theorem, given by reviewer and R. Prim [unpublished report], states that any region of such an irrotational flow, steady or not, is a region of constant entropy or a region where pressure and density of each particle remain constant, provided the extraneous force is conservative. When assumptions of steady motion and vanishing extraneous force are added, it follows that the speed is constant upon each streamline along which pressure is constant, and hence by a theorem of Hamel [*S.-B. Preuss. Akad. Wiss.* **1937**, 5–20] that any region in which grad $\eta \neq 0$ is a region of helicoidal flow. Author is concerned with proving that, with certain specific exceptions, an irrotational flow under these assumptions cannot consist of some portions in which grad $\eta = 0$ joined to other portions in which flow is helicoidal. He proves, first, that boundary between two such portions must necessarily be a streamline belonging to the one or the other, and, second, that if a streamline contains one boundary point of region of helicoidal flow, all its points are boundary points, it is itself a circular helix extending to boundary of flow, and pressure, density, and speed are constant upon it. Henceforth, he limits his attention to plane and rotationally symmetric flows, and bases his results upon uniqueness theorems applicable to partial differential equations for velocity potential in the two cases. These uniqueness theorems concern Cauchy data; for cases when flow upon boundary curve is subsonic or supersonic, known results concerning elliptic and hyperbolic equations respectively are applied, but for case of a boundary upon which speed is sonic a new uniqueness theorem for parabolic data valid except when curve is straight is proved by author, who conjectures that it holds also for straight boundaries. By applying the uniqueness theorems for subsonic case, author proves that a plane irrotational flow which is subsonic throughout a region is either a flow of uniform entropy or a simple vortex throughout the region, and that a rotationally symmetric irrotational flow which is subsonic in a region is either a flow of uniform entropy or a uniform flow throughout. He gives the counterexample of a uniform flow of variable entropy having part of a straight streamline in common with a uniform flow of constant entropy continued into a corner flow to show that these theorems cannot be extended without exception to supersonic case. He then defines a noncusped streamline as any streamline which is not an infinite half-line, a circular arc, a closed curve consisting in part of a circular arc with a cusp at each end of the arc, nor a curve consisting in part of a half line with a cusp at the end of a half line, and proves that above theorems hold for purely supersonic flows in regions bounded by noncusped streamlines. In case of mixed flows, the theorems so stated would continue to hold if author's conjectured uniqueness theorem for parabolic data were admitted, but with the somewhat weaker theorem he was able to prove he obtains the result that if a plane irrotational flow in a region bounded by noncusped streamlines is not a flow of uniform entropy, and if further the portion of flow in which grad $\eta \neq 0$ contains a curved streamline, then the flow is a simple vortex. Author states that if uniqueness theorems stronger than those now known were available for partial differential equations in three independent variables, the results could be extended to flows in three dimensions. C. Truesdell, USA

Turbulence, Boundary Layer, etc.

(See also Rev. 490)

342. **Holstein, H., On the outer and inner boundary layer for oscillations of laminar flow** (in German), *Z. angew. Math. Mech.* **30**, 1/2, 25–49, Jan.–Feb. 1950.

Author, using Tollmien's method, develops asymptotic formulas for the main stability functions and investigates the limits of these expressions in various regions of integration. The functions ϕ_3', and ϕ_3 are obtained by integration of the asymptotic expressions for ϕ_3''. The values of Tietjen's function ($= \phi_3/\phi_3'$) are found to be in good agreement with results of Lin and others.

The function ϕ_2, in the neutral case $c_i = 0$, is also found and the results compare favorably with, but are more accurate than, those due to Schlichting. Finally, the calculation of ϕ_2 is made for both very small and very large values of positive and negative c_i. These latter calculations show that a single "inner boundary layer" exists in all cases and it vanishes only for finite large positive and negative c_i. This is contrary to results of Lin, who showed that two inner boundary layers exist in case of damped oscillations. Author criticizes Lin's deductions on basis of his trying to find a physical interpretation for the limits of the asymp-

totic development of the integration formulas [see also Wasow, W., *Annals Math.* **29,** 1948]. G. M. Lilley, England

343. Corrsin, Stanley, Hypothesis for skewness of the probability density of the lateral velocity fluctuations in turbulent shear flow, *J. aero. Sci.* **17,** 7, 396–398, July 1950.

The skewness in the probability density of transverse velocity fluctuations in a turbulent shear flow is explained by the corresponding lateral gradient in lateral turbulence intensity. A linearized estimate is deduced by introducing a characteristic length Λ. Reviewer thinks it interesting that in both numerical examples Λ is of same magnitude as the old mixing length, though definitions and meanings of these two lengths are different.

Karl Wieghardt, England

344. Eckert, Hans Ulrich, Characteristics of the turbulent boundary layer on a flat plate in compressible flow from measurements of friction in pipes, *J. aero. Sci.* **17,** 9, 573–584, Sept. 1950.

The Prandtl-von Kármán method (turbulent, incompressible flow) is systematically extended to compressible flow. Assuming shear stress at the plate is given by that at the pipe wall when pipe-flow values and radius are interpreted as b. l. (boundary layer) edge values and thickness respectively, the thickness then follows from von Kármán's momentum integral relation if a velocity profile is assumed. Other assumptions: (1) Variables represent time averages. (2) Turbulent b. l. starts from leading edge. (3) Velocity profiles are homologous, given by a power law with constant exponent, and are the same for pipe and plate. (4) Pressure does not vary normal to flow. (5) For compressible flow and insulated boundaries, constant kinetic energy plus enthalpy per unit mass relates velocity and temperature. (6) Sutherland's viscosity-temperature relation can be used, but for simplification $\mu \sim T^{\omega}$ may apply ($\omega = 0.8$ in this work).

Combinations of variables including displacement and momentum thickness ratios are expressed in terms of mean density (over normal coordinate) and velocity profile exponent. They are given graphically for M (Mach number) from 0 to 10 and various profile exponents. Pipe tests are cited to show that compressibility affects pipe friction coefficients only by modifying R_e (Reynolds number). Hence Blasius' friction law implying a seventh-root velocity profile is used to obtain the b. l. thickness. Results show b. l. thickness increases to 1.32 times incompressible, and friction coefficient decreases to 0.68 of incompressible at $M = 4$. Simplified analysis is considered valid for $0 < M < 4$ and a range of R_e from 6 times Lees' critical stability value to about 10^7. Quantitative agreement with other theories is not close, but preliminary experiments verify the assumptions for $M < 2$. Author remarks that similar results can be obtained without pipe tests by basing the incompressible flow equations on some mean values for viscosity and density.

Martin Bloom, USA

345. Laufer, John, Some recent measurements in a two-dimensional turbulent channel, *J. aero. Sci.* **17,** no. 5, 277–287, May 1950.

Detailed measurements are presented of the mean and turbulent velocities, correlation coefficients, and microscales in turbulent flow between parallel walls. Value of this work lies in fact that it represents one of the few really complete studies of a non-isotropic flow field. Flow studied was relatively simple in not varying in the flow direction. Hot wires in various sizes and orientations were used for the measurements in a 5-in. wide channel (aspect ratio 12) at three Reynolds numbers. A few measurements are mentioned in a 1-in. channel (aspect ratio 60).

Mean velocity distributions are found to follow the logarithmic relation except within the laminar film, whose extent is in agreement with usual predictions. The dimensionless velocity fluctuations, correlation coefficient k, and microscales decrease with increasing Reynolds number. At the wall the ratio of the rms velocity fluctuation in flow direction to the mean velocity u'/u tends to approach a finite value independent of Reynolds number. The microscales have a slight but consistent maximum at the wall distance $y/d = 0.7$. The spectrum measurements of the u' fluctuations indicate an n^{-7} (n = frequency) variation in high frequency region. Comparison of spectra of velocity fluctuations and of Reynolds stresses indicates local isotropy above 1500 cps.

J. M. Robertson, USA

346. Laufer, John, Investigation of turbulent flow in a two-dimensional channel, *Nat. adv. Comm. Aero. tech. Note* 2123, 67 pp., July 1950.

Paper is a more detailed version of study reported in the preceding review. Only the important chief omissions from the first paper will be reviewed.

Measurements in the 1-in. channel are presented in detail. Comparison with the 5-in. channel results, at same Reynolds number, indicates certain anomalies. Turbulent shear distributions across the 5-in. channel were obtained by direct and indirect methods. Except for highest Reynolds number, the three methods are in agreement. A plot of the rms longitudinal velocity fluctuations divided by the friction velocity as a function of the friction distance parameter, results in similar curves within laminar film. Typical R_x and R_y correlation curves are given together with values of length scales determined therefrom. A comparison of values of the correlation coefficient k obtained in this and studies by others indicates a slow decrease in maximum value with increase in Reynolds number. It also appears that other parameters are important, thus k tends to increase with a decrease in channel width.

The energy balance in the turbulent field, where direct viscous dissipation is negligible, was calculated from the measured quantities. This results in a picture of the energy being produced in a wall region extending out to about $y/d = 0.7$, most of it being dissipated there by breaking down of large eddies into smaller ones. Energy dissipated in the central region is mostly diffused into it. A constant rate of energy dissipation in the central region is indicated by constancy of the microscales.

J. M. Robertson, USA

347. Chandrasekhar, S., On Heisenberg's elementary theory of turbulence, *Proc. roy. Soc. Lond. Ser. A* **200,** no. 1060, 20–33, 1949.

The distribution of turbulent energy with respect to frequency is considered on the basis of the concepts developed by Heisenberg (*Z. Phys.* **124,** p. 628; *Proc. roy. Soc. A* **195,** p. 402), whose equations are solved explicitly for cases of (a) steady-state distribution arising from a steady source of turbulent energy, and (b) unsteady-state decay of turbulence initially characterized by the steady-state distribution. In case (a) the fraction of the total kinetic energy of turbulence, $F(k)dk$, found among eddies having wave numbers within range dk is given by $F(k) = F_0(k_0/k)^{5/3}\{1/[1 + (k/k_s)^4]^{4/3}\}$, where F_0 and k_0 are constants, and k_s is the wave number at which total rate of dissipation of energy by molecular viscosity is of same order as rate of transfer of turbulent energy from larger eddies to smaller ones. k_s is proportional to the $^3/_4$ power of Reynolds number, defined as the product of the rms deviating velocity times the diameter of largest eddies present, divided by the kinematic viscosity. In absence of any fluid viscosity ($k_s = \infty$), result reduces to Kolmogoroff's equilibrium spectrum [*C. R. Acad. Sci. URSS* **30,** p. 301, 1941; **32,** p. 16, 1941].

In case (b) the mathematical results are more complex, but calculated values of a frequency distribution function, which maintains its general shape as the large eddies transfer their energy to smaller ones, are tabulated for various values of the unchanging Reynolds number of the turbulence.

R. L. Pigford, USA

348. Tetervin, Neal, and Lin, Chia Chiao, **A general integral form of the boundary-layer equation for incompressible flow with an application to the calculation of the separation point of turbulent boundary layers,** *Nat. adv. Comm. Aero. tech. Note* 2158, 63 pp., Aug. 1950.

Paper derives an integral form of boundary-layer equation by multiplying each term of this differential equation by u^m and, after writing in a suitable form, once more by y^n and then integrating from $y = 0$ to $y = \delta$. The resulting general integral form contains the Loĭtsyanskiĭ equation (for $n = 0$), the Wieghardt equation of kinetic energy ($m = 1$, $n = 0$) and the von Kármán momentum equation ($m = 0$, $n = 0$) as special cases. By using a single parameter family of velocity profiles (power profiles), this general equation is changed to an equation for the rate of change of the profile parameter. As by other authors, it is found it is necessary that additional knowledge be found concerning the surface shear and the distribution of shearing stress across turbulent boundary layer. Reviewer's remark: The increase of surface shear before separation point, cited by authors, has been refuted in the meantime by Ludwieg and Tillmann [see *Ingen.-Arch.* **XVII**, p. 288, 1949, or *NACA T. M.* 1284 and 1285].

F. W. Riegels, Germany

349. Townsend, A. A., **The eddy viscosity in turbulent shear flow,** *Phil. Mag.* (7), **41**, 320, 890–906, Sept. 1950.

Continuation of previous work of author on the structure of the turbulent wake of cylinder. Contribution to basic understanding of turbulent shear flow, further demonstrating falsity of usual assumptions of mixing length theory. Spectrum functions of velocity fluctuation components measured 500 diam downstream from $^1/_{16}$-in. cylinder at 42 ft/sec. These show typical characteristics due to superposition of two groups of turbulence of widely different scales, i.e., large-scale eddies mainly responsible for convective mixing and small-scale eddies containing most of the turbulent energy. The properties, structure, and role in energy transfer of the large eddies are discussed in detail. It is suggested that the large eddies are in dynamic equilibrium. The effect of the small eddies may be described by the concept of eddy viscosity, but eddy viscosity is not related to local parameters of mean flow. Suggestions toward a new theory and relevant experiments are described.

H. L. Dryden, USA

350. Batchelor, G. K., **Note on free turbulent flows with special reference to the two-dimensional wake,** *J. aero. Sci.* 17, 7, 441–445, July 1950.

The mixing length theory of turbulent transfer is examined with reference to its lack of internal consistency and its inability to describe the actual structure of the turbulence in wake flow. Recent experimental work at Cambridge shows that the flow is divided, fairly sharply, into a region of almost laminar flow and a wholly turbulent wake core with an irregular boundary, within which the turbulence has small-scale isotropy and similarity. Within the core, the turbulent energy is nearly uniform, and the turbulence is roughly divisible into the small eddies comprising the core, and large eddies that distort the core boundary. Corresponding to these two distinct groups of eddies, two types of transfer may co-exist: gradient diffusion by the small eddies, and bulk convection by the large eddies. Due to its peculiar nature, only the former process is effective for momentum transfer, but both processes transport turbulent energy, heat, etc. An analytical treatment of this general mechanism of diffusion has yet to be formulated.

A. A. Townsend, England

351. **Corrsin, Stanley, and Uberoi, Mahinder S., Spectrums and diffusion in a round turbulent jet,** *Nat. adv. Comm. Aero. tech. Note* 2124, 80 pp., July 1950.

In a round turbulent jet at room temperature, measurement of the shear correlation coefficient as a function of frequency (through band-pass filters) has given a rather direct verification of Kolmogoroff's local isotropy hypothesis. This is concluded from the monotonic decrease to zero of the shear-correlation spectrum ($\overline{u_n v_n}/u_n' v_n'$) with increasing frequency n.

One-dimensional power spectrums of velocity and temperature fluctuations, measured in unheated and heated jets, respectively, have been contrasted. Under the same conditions, the two corresponding transverse correlation functions have been measured and compared. Measurements have been made of the mean thermal wakes behind local (line) heat sources in the unheated turbulent jet, and the order of magnitude of temperature fluctuations has been determined.

M. Schäfer, Germany

352. **Schubauer, G. B., and Klebanoff, P. S., Investigation of separation of the turbulent boundary layer,** *Nat. adv. Comm. Aero. tech. Note* 2133, 57 pp., Aug. 1950.

Paper presents an extensive set of measurements of characteristics of a turbulent boundary layer near a smooth surface at a high Reynolds number in incompressible flow with pressure gradients sufficient to cause flow separation, namely pressure, velocity profiles, turbulence intensities, turbulent shearing stress τ, transverse and longitudinal correlations. Separation occurred at a shape factor $H = 2.7$. Results support the idea that the concept of energy transfer through a spectrum, as known for isotropic turbulence, may be extended to the turbulent boundary layer. Fediaeosky's theory of distribution of τ is partly in excellent, partly in poor agreement with experimental results; Nevzgljadov's theory is not supported at all, as it seems that correlation between u and v components depends on the mean velocity gradient as a whole rather than upon the local.

H. Wijker, Netherlands

353. **Burgers, J. M., The formation of vortex sheets in simplified type of turbulent motion,** *Proc. Ned. Akad. Wet.* **53, 2, 122–133, Feb. 1950.**

One case of turbulent motion is that of vortexes with axes in direction of flow. This case is extended by a purely mathematical analysis of the formation of vortex sheets. Equations presented for a simplified model may help in understanding certain features of turbulent flow.

R. C. Binder, USA

354. **Shvets, M. E., Heat transfer in the laminar boundary layer on a body of revolution** (in Russian), *Prikl. Mat. Mekh.*, 102–104, Jan.–Feb. 1950.

Author develops an iterative method for determining velocity and temperature profile in boundary layer on bodies of revolution. As zero approximation, linear profiles both for velocity and temperature are taken, and inserting them into the boundary layer equations, approximations of higher order are obtained. Condition that velocity and temperature gradient approach zero for great distance from wall supplies two differential equations for thicknesses of the velocity and temperature boundary layers. Method, identical in principle with the Pohlhausen method, is applied to problems where the velocity and temperature outside of the boundary layer follow power laws. Results

for different Prandtl numbers are found to agree with exact solution of Falkner and Fage. Walter Wuest, Germany

355. Thwaites, B., Approximate calculation of the laminar boundary layer, *Aero. Quart.* **1,** 245–280, Nov. 1949.

Known velocity distributions from exact and approximate solutions of the plane-flow boundary-layer equations are collected. These include, besides the Pohlhausen family and Blasius distribution, a number of cases of both accelerated and decelerated flow and one case of distributed suction. They are compared by plotting curves of $l(m)$ and $H(m)$, where $(\partial u/\partial y)_{y=0} = Ul/\theta$, $(\partial^2 u/\partial y^2)_{y=0} = Um/\theta^2$, θ denotes momentum thickness, H ratio of displacement to momentum thicknesses, $U(x)$ potential velocity outside the boundary layer, x, y usual boundary-layer coordinates, and u, v corresponding velocity components. It is found that there is remarkably little difference between the various curves of $l(m)$ and of $H(m)$ for practical values of m. Proposed method of approximate calculation is based on use of average curves; these are drawn so as to give correct values at points representing Blasius' case, separation, stagnation point, and suction (asymptotic behavior).

Knowledge of $l(m)$ and $H(m)$, together with von Kármán's momentum equation, is sufficient to permit calculation of boundary-layer thickness. New method is tested in two cases: first is Howarth's case of linearly decreasing pressure gradient, second is Schubauer's elliptic cylinder. Numerical accuracy is good in comparison with exact solutions. W. R. Sears, USA

356. Timman, R., A one-parameter method for the calculation of laminar boundary layers, *Nat. Luchtlab. Amsterdam Rap.* **F. 35,** F 29–45, 1949.

This is an application of von Kármán momentum equation to prediction of laminar boundary-layer behavior in a pressure gradient. The new family assumed for velocity profiles is suggested by the details of von Kármán-Millikan solution (*NACA T. R.* 504, 1934).

Results computed for comparison with Schubauer's ellipse measurements (*NACA T. R.* 527, 1935) show some improvement over Pohlhausen method (ZAMM **1,** 1921). Unfortunately, it is found necessary to use different boundary conditions for accelerated and for retarded flow.

Stanley Corrsin, USA

357. Batchelor, G. K., and Townsend, A. A., The nature of turbulent motion at large wave-numbers, *Proc. roy. Soc. Lond. Ser. A* **199,** 238–255, Oct. 1949.

Nature of spectrum of turbulence for large wave numbers is considered from theoretical and experimental standpoints. For this purpose the ratio D_6D_2/D_4^2 where $D_{2n} = \overline{U_1^2}\partial^{2n}f(r)/\partial r^n$ where $f(r) = \overline{U_1(x)U_1(x+r)}/\overline{U_1^2}$, is the correlation coefficient of velocity components $U_1(x)$ and $U_1(x+r)$ at x and $x+r$ in the x direction. Ratio D_6D_2/D_4^2 is related to spectrum of turbulence $F(k)$, by

$$\frac{D_6D_2}{D_4^2} = \frac{35}{27}\frac{{}_0\int^\infty k^6F(k)dk\ {}_0\int^\infty k^2F(k)dk}{[{}_0\int^\infty k_4F(k)dk]^2} \qquad [*]$$

It is known that for small Reynolds numbers, $F(k) = \text{const}\cdot ke^{-2\nu tk^2}$ and [*] gives 1.94 for ratio under consideration. On the other hand, Kolmogoroff's theory of locally isotropic turbulence leads one to expect that the ratio D_6D_2/D_4^2 will tend to a limiting value for large Reynolds numbers and independently of circumstances under which turbulence is initiated. These predications are confirmed experimentally, and it is further found that a value exceeding four for ratio is reached in limit of large Reynolds numbers. This high value is in agreement with the k^{-7} spectrum law predicted by Heisenberg on an elementary theory, provided that this spectrum is cut off at some value k_R. Experiments suggest that corresponding critical Reynolds number is of order of 0.0045. According to authors, the high value of D_6D_2/D_4^2 indicates a state of affairs which has not been contemplated in theories of turbulent motion. As an explanation of this, authors suggest that "the distribution of vorticity (and of higher-order derivations) is *made* 'spotty' in early stages of decay by some intrinsic instability, and *kept* spotty throughout decay by action of quadratic terms of Navier-Stokes equations."

S. Chandrasekhar, USA

358. Agostini, L., and Bass, J., Theory of turbulence (Les théories de la turbulence) (in French), Paris, *Publ. Sci. Tech. Ministère de l'Air,* no. 237, 1950, viii + 118 pp.

Book provides a unified exposition of statistical theory of turbulence as it has been developed in recent years, starting with work of G. I. Taylor and ending about middle of 1949. It begins with a chapter on random functions and their use in describing turbulent motion. Chapter 2 describes various correlation and spectral functions, special forms which they assume when assumptions such as homogeneity, isotropy, and incompressibility are made, and relations between the different functions. In chapter 3 the implications of Navier-Stokes equations are discussed. Various similarity hypotheses are treated in a unified fashion. Assumptions underlying Heisenberg's work are discussed and his fundamental equations are derived. In chapter 4 Kolmogorov's similarity theory for locally isotropic turbulence and its relation to the Weizsäcker-Heisenberg theory are discussed. Last chapter takes up the problem of decay of turbulence behind a grid, in particular, the work of Batchelor and Townsend on initial and fixed phases of decay. Work concludes with a list of fundamental definitions and formulas, some experimental curves obtained by Favre, and a short but adequate bibliography (although the papers of Loĭtsyanskiĭ and Obuhov are missing). In general, book gives a rather clear summary of present situation in this field. J. V. Wehausen, USA

359. Chandrasekhar, S., The theory of statistical and isotropic turbulence, *Physical Rev.* (2) **75,** 896–897, 1949.

Author obtains an explicit formula for spectrum of turbulence at high frequencies from an equation of Heisenberg, who gave only an interpolation formula. Present formula is $F(k) = F(k_0)(k_0/k)^{5/3}[1 + (k/k_s)^4]^{-4/3}$, where $F(k)$ is the spectrum and k_0 and k_s are two typical wave numbers ($k_0/k_s \ll 1$). [Reviewer's remark: Essentially same result has been obtained independently by J. Bass.] C. C. Lin, USA

360. Watson, E. J., Asymptotic solution of a boundary layer suction problem, *Aero. Res. Counc. Rep. Mem.,* R. & M. 2298, July 1946, 8 pp., 1950.

Theory of the boundary layer on a flat plate in a uniform stream with a velocity of suction proportional to $x^{-1/2}$ (x being distance from leading edge of plate), has been developed by Thwaites in a report which contains numerical solutions of the problem obtained on differential analyzer. Behavior of the solution when rate of suction is large is investigated here, and it is found that velocity distribution in boundary layer approximates to the Griffith-Meredith or asymptotic suction profile. Solution is developed in form of a series of descending powers of the suction velocity, and coefficients of this series are obtained by solution of linear differential equations. First four coefficients are obtained explicitly and numerical values are given. Series are also obtained for displacement and momentum thicknesses and for skin

friction and form parameter H. Comparisons are made with Thwaites' solutions, and good agreement is found when rate of suction is large.
From author's summary

361. **Kawahara, Takuma, The theory of the turbulent boundary layer and its application** (in Japanese), *J. Jap. Soc. appl. Mech.* **3**, 1–5, 1950.

Defining the turbulent boundary layer as the domain where eddies predominate, author determines its thickness by solving the equations which are derived from von Kármán's fundamental dissipation equations of the mean eddy energy and vorticities, and compares results with the well-known relations.
From author's summary

Aerodynamics of Flight; Wind Forces

(See also Revs. 188, 466)

362. **Gardiner, Robert A., Zarovsky, Jacob, and Ankenbruck, H. O., An investigation of the stability of a system composed of a subsonic canard airframe and a canted-axis gyroscope automatic pilot,** *Nat. adv. Comm. Aero. tech. Note* 2004, 51 pp., Jan. 1950.

Standard technique of linear servomechanism theory is used to determine the stability of the closed-loop system by noting behavior of the open-loop frequency response plotted in a Nyquist diagram. Transient responses of the air frame are calculated for both longitudinal and lateral cases. While the methods used are clearly valid when system is linear, reviewer believes that insufficient attention has been paid to fact that the measured autopilot characteristics shown are nonlinear and that the results are, at best, qualitative [cf. *NACA T.N.* 1901, June 1949; Rev **3**, 1758].
John Wild, USA

363. **Miles, John W., Unsteady flow theory in dynamic stability,** *J. aero. Sci.* **17**, 1, 62–63, Jan. 1950.

An argument for the importance of unsteady flow effects on the damping terms in the dynamic stability equations.
Harvard Lomax, USA

364. **Gates, S. B., Note on the relation between turning and straight performance,** *Aero. Res. Counc. Rep. Mem.*, R. & M. 2382, 7 pp., 1950.

In R. & M. 2381, a diagrammatic method is developed for estimating turning performance when straight performance is known, assuming a parabolic variation of drag coefficient with lift coefficient. Using same assumption, theory has now been re-stated so as to indicate explicitly the relation between turning and straight performance; some general conclusions follow. This leads to a less laborious method of estimating turning performance, which seems very suitable for rapid prediction as a natural extension of straight performance investigations at the design stage. An example is worked out, and an approximation is suggested which indicates, roughly, general features of deterioration in turning performance as the ceiling is approached.
From author's summary

365. **Seamans, R. C., Jr., Blasingame, B. P., and Clementson, G. C., The pulse method for the determination of aircraft dynamic performance,** *J. aero. Sci.* **17**, 1, 22–38, Jan. 1950.

Aim of paper is to show that dynamic performance of aircraft can be conveniently determined from flight tests by finding response of a component to a pulse, applied by pilot. Here the oscillations will be control-fixed. The mathematical theory is first given of determining response of a component to a given forcing function, particularly to a triangular pulse. Use of a "performance function," equal to ratio of Fourier transform of output to transform of input, is shown to be convenient here. Practical methods are explained for determining the performance function from pulse response data for a damped oscillating system. Use of a Fourier synthesizer and analyzer is recommended. Method of making a pulse test is described in some detail, and it is concluded that use in flight tests of pulse method, as distinguished, for example, from step displacement or sinusoidal tests, leads to a saving in equipment, installation time, and flying time.
M. Morduchow, USA

366. **Luskin, H., Predictions of supersonic airplane performance,** *J. aero. Sci.* **17**, 1, 5–12, Jan. 1950.

A brief discussion is first given of growth curves and particularly of growth of airplane speeds. Author points out that new developments in jet aircraft may strongly change present rate of growth of airplane speeds. Brief analysis is given of subsonic and supersonic airplane performance, including ceiling and wing loading for maximum L/D. Author shows, for instance, that ceilings of over 50,000 ft can be attained by supersonic aircraft. Higher wing loadings for the future are shown to be expected, together with higher (supersonic) top speeds, cruising speeds, and ceilings. Improvement in flying safety is also anticipated.
M. Morduchow, USA

367. **Stewart, Wm., and Burle, M. F., The application of jet propulsion to helicopters,** *Curr. Pap. aero. Res. Counc. London*, no. 8, 12 pp., 1950.

Author compares several methods of applying principle of jet propulsion to helicopter rotor blades and includes an analysis of a ram-jet system and a pressure-jet system, giving fuel consumption and performance information of hypothetical helicopters powered by these two power plants. These performance data are graphically compared with a conventional helicopter.

There is a brief discussion of methods of applying jet propulsion to helicopters including direct replacement of reciprocating engine by a gas turbine; using a gas turbine at blade tips; using ram jets at blade tips; and using an air compressor in fuselage to pressurize a jet stream flowing out the blade with combustion taking place at the blade tips. Of systems described, ram-jet is the most simple form.

It is shown in paper that, for a helicopter of given gross weight, the ram-jet configuration is capable of higher pay load for a short duration. Pressure-jet system operates as a compromise between ram jet and conventional helicopter. Although optimum operating condition for each configuration has been developed, this is a function of so many variables such as blade solidity, rotor tip speed, etc., that any specific project would have to be considered in detail before a choice of configuration could be made. It is noted that author has neglected to include consideration of pulse-jet power plant, for reasons unknown. The payload endurance performance curves shown would be more complete if pulse-jet system were included. R. A. Young, USA

368. **Williams, D. H., On the integrating comb method as applied to profile-drag results in the compressed air tunnel,** *Aero. Res. Counc. Rep. Mem.*, R. & M. 2318, 7 pp., 1950.

Drag measurements obtained by pitot traverse in the compressed air tunnel are compared with those that would have been given by an integrating comb and with those given by generalized curves due to Young (R. & M. 2257). Agreement is good in general, but as errors due to using approximate methods can be

anything up to 3%, measurements would not be accurate enough to determine small differences in drag coefficients.

From author's summary

369. McCraig, I. W., Water stability tests on Saro 37 fitted with Shetland hull bottom, wing tip floats and tail, *Curr. Pap. aero. Res. Counc. London*, no. 3, 16 pp., 1950.

Tests have been made on the Saro 37 with a Shetland hull bottom, wing-tip floats and tail, to find water stability limits over a range of weights and C. G. positions. Free-to-trim attitude and elevator effectiveness were also measured.

From author's summary

370. Morris, J., Jones, W. A., and Walton, G. F., Axial and longitudinal vibration of an aircraft, *Aero. Res. Counc. Rep. Mem.*, R. & M. 2289, 24 pp., 1950.

Report gives an analytical solution for axial or longitudinal vibration of a single-engined aircraft. Solution is applied numerically to a particular fighter for which some experimental observations were available. Treatment is based on replacement of the continuous mass system by a finite system of discrete masses elastically interconnected. In course of analysis, use is made of deflection coefficient method in formulation of equations of motion and escalator process in their marshalling and numerical solution. Agreement between theoretical and observed results appears satisfactory. From authors' summary

371. Press, Harry, The application of the statistical theory of extreme values to gust-load problems, *Nat. adv. Comm. Aero. tech. Note* 1926, 43 pp., Nov. 1949.

Report is an example of the benefits obtained from fitting experimental data into the best possible theoretical framework. Up to this time, it has been general practice to consider data obtained from gust-load investigations as random samples for conditions studied. Because of the arbitrary nature of the distribution curves fitted to data, extrapolated predictions were of poor reliability. Report applies recent developments in statistical theory of extreme values to gust-load data. It is shown that such estimates of frequency of encountering larger loads are more consistent with data, and more reliable than those obtained in previous analyses. W. N. Turner, USA

372. Stevens, G. W. H., and Johns, T. F., The theory of parachutes with cords over the canopy, *Rep. Memo. aero. Res. Counc. London*, R. & M. 2320, 10 pp., 1949.

Stress distribution in a parachute with cords over the canopy is investigated, especially with cords shorter than length of the fabric gore which, under these conditions, bulges out between cords. An approximate theory of shape and stress distribution is developed. It is assumed that fabric tension in any axial direction can be reduced to a negligible amount and that net pressure distribution over the parachute is uniform. Although assumed shapes do not quite agree with actual shapes, the practical correctness of the theoretical load equation is said to be substantiated by tests. H. P. Leipman, USA

373. Kaufman, William M., Liddell, Charles J., Jr., Smith, Allan, and Van Dyke, Rudolph D., Jr., An apparatus for varying effective dihedral in flight with application to a study of tolerable dihedral on a conventional fighter airplane, *Nat. adv. Comm. Aero. Rep.* 948, 16 pp., 1949.

An apparatus for varying effective dihedral in flight by means of servo actuation of the ailerons in response to sideslip angle is described. Results of brief flight tests of the apparatus on a conventional fighter airplane are presented and discussed. Apparatus is shown to have satisfactorily simulated a wide range of effective dihedral under static and dynamic conditions. Effects of a small amount of servo lag are shown to be measurable when apparatus is simulating small negative values of dihedral. However, these effects were not considered by pilots to give airplane an artificial feel.

Results of an investigation employing the apparatus to determine tolerable (safe for normal fighter operation) range of effective dihedral on test airplane are presented. A survey of pilots' opinions was made to determine which values of effective dihedral were intolerable. It was found that small amounts of negative dihedral (of the order of $-5°$) as well as values of positive dihedral greater than 20° could be tolerated by the pilots; in fact, at landing-approach speeds an effective dihedral high enough (28.4°) to produce oscillatory instability could be tolerated. Occurrence of rolling-velocity reversals during rudder-fixed aileron rolls with high positive values of effective dihedral did not adversely affect the pilots' opinions of the over-all lateral handling characteristics. Relation between findings of this investigation and present Air Force-Navy stability and control specifications is discussed. From authors' summary by E. A. Bonney, USA

374. Ellis, Macon C., Jr., and Hasel, Lowell E., Preliminary investigation at supersonic speeds of triangular and sweptback wings, *Nat. adv. Comm. Aero. tech. Note* 1955, 37 pp., Oct. 1949.

A series of thin, triangular plan-form wings have been investigated. Series consisted of eight triangular wings of vertex angles such that a range of leading-edge positions both inside and outside Mach cone at two test Mach numbers of 1.43 and 1.71 was obtained. Three sweptback wings having angles of sweep of 45°, 55°, and 63° were also tested at a Mach number of 1.43.

Lift results for triangular wings indicated that Jones' theory for lift of slender pointed wings is applicable for thin wings in the range of test Mach numbers up to values of $\tan \epsilon/\tan m \approx 0.3$, where ϵ is wing vertex half angle, and m is Mach angle. Center of pressure of triangular wings was coincident with center of area for all wings tested at both Mach numbers. Lowest minimum drag coefficients were obtained for wings with smallest vertex angles relative to Mach angle. Also, in this smallest vertex-angle region, highest values of maximum lift-drag ratio of about 7 for both Mach numbers were obtained.

Andrew Vazsonyi, USA

375. Legras, Jean, Contribution to the study of wings (in French), *Publ. Sci. Tech. Ministère de l'Air*, Paris, no. 222, v + 93 pp., 1949.

After two introductory chapters, author attacks problem of thin sweptback wing in incompressible flow. He writes the induced velocity as the sum of a number of terms, but seems to leave most troublesome of these unsolved, and proceeds to the circular wing. He shows that any lifting-surface problem can be decomposed into one without circulation and a pure circulation required to satisfy edge conditions. Therefore, he first treats the circular wing without circulation. Two types of potential functions are found, related to Legendre polynomials, and a solution in series is proposed. Suitable potentials for the circulatory part are then derived from these, and the solution is formally complete.

W. R. Sears, USA

376. Coombs, A., Notes on the forces acting on a two-dimensional aerofoil in shear flow in the presence of a plane boundary, *Proc. Cambr. philos. Soc.* **45**, 612–620, Oct. 1949.

Equations are given which can be used to calculate lift and

pitching moment of a two-dimensional airfoil of general shape situated in an incompressible nonviscous stream that has a constant velocity gradient in direction normal to stream velocity and is either unbounded or bounded on one side by a plane. In the analysis, the stream function is written as sum of stream function of undisturbed flow and of a disturbance stream function. Although undisturbed flow is rotational, motion represented by disturbance stream function is irrotational; thus, methods of complex variable theory are applicable. Shear-flow problem then becomes one of finding a complex potential function that satisfies the given boundary conditions.

As a special case of the general airfoil, equations for and figures of variation of lift with ratio of chord of a plate to its distance from ground are given for a flat plate at an angle of attack near ground. Formulas and figures for lift, pitching moment, and center of pressure location are also given for a flat plate at various angles of attack with its trailing edge in contact with the ground.

Neal Tetervin, USA

377. **Cahill, Jones F., Summary of section data on trailing-edge high-lift devices,** *Nat. adv. Comm. Aero. Rep.* 938, 30 pp., 1949.

A summary has been made of available data on characteristics of airfoil sections with trailing-edge high-lift devices. Data for plain, split, and slotted flaps are collected and analyzed. Effects of each of the variables involved in design of various types of flap are examined and, in cases where sufficient data are given, optimum configurations are deduced. Wherever possible, effects of airfoil section, Reynolds number, and leading-edge roughness are shown. For single- and double-slotted flaps, where a large amount of unrelated data are available, maximum lift coefficients of many configurations are presented in tables.

From author's summary

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 328, 370)

378. **Farquharson, F. B., Aerodynamic stability of suspension bridges, part I,** *Bull. Univ. Wash. engng. Exp. Sta.* no. 116, 103 pp., 1950.

Part 1 deals with history of dynamic behavior of suspension bridges, including Tacoma Narrows Bridge, aerodynamic theories entertained during early studies, and model testing. A detailed account, including photographs, is given of behavior of Tacoma Narrows Bridge during its brief life. A brief description is given of the so-called autorotation theory, in which instability results when the suspended structure has a negative slope of lift or moment versus angle of attack. A short discussion of the problem of model similitude is given, together with descriptions of a static erection model and a three-dimensional dynamic model built for Tacoma Narrows Bridge. Results are given of static wind tunnel tests with various types of girders on the suspended structure. These tests indicate that the autorotation theory is inadequate in explaining the vertical oscillations observed on bridge since slope of lift curve is positive on girder sections of the Tacoma proportions. However, slope of moment curve is unquestionably negative, suggesting that torsional motions may have their origin in autorotation. Author states that evidence is available to indicate that autorotation theory alone is probably inadequate.

R. L. Bisplinghoff, USA

379. **Steinman, D. B., Aerodynamic theory of bridge oscillations,** *Proc. Amer. Soc. civ. Engrs.* **75,** 8, 1147–1184, Oct. 1949.

Equations of motion of an oscillating nonstreamline section (e.g., a bridge section) in a uniform air flow are set up for uncoupled and for coupled vertical and angular motions. Equations are applied to an H-section having proportions close to those of Tacoma Narrows Bridge. Using aerodynamic data obtained from static tests on inclined and on curved wind tunnel models, analytical results obtained for vertical and for torsional instability agree with observations at the bridge and with oscillating model wind tunnel tests. Validity of using inclined static models to simulate vertically-moving sections and of curved static models to simulate rotating sections in a uniform air flow has thus been verified for bridge oscillation problems.

Walter W. Soroka, USA

380. **Germain, P., and Bader, R., Some remarks on the vibratory motions of a wing in a supersonic regime** (in French), *Rech. aéro. Paris,* no. 11, 3–13, Sept. and Oct. 1949.

Disturbance velocity potential for a harmonically oscillating wing flying at a supersonic Mach number M can be calculated from the equation

$$\varphi_{xx} - \beta^2(\varphi_{yy} + \varphi_{zz}) + k^2\varphi = 0.$$

The potential is $\varphi \exp\{i\omega(t - xV/\alpha^2\beta^2)\}$ where x, y, z are usual coordinates, t time, $\beta^2 = M^2 - 1$, V speed of flight, $\alpha = \sin^{-1}(M^{-1})$, ω circular frequency, and $k^2 = (\omega^2 \sin^2\alpha)/(V^2\cos^4\alpha)$. If the wing surface is defined by $z = g(x, y)\exp(i, \omega t)$, the boundary condition at the wing is, approximately,

$$(\varphi_z)_{z=0} = (Vg_x + i\omega g)\exp(i\omega xV/\alpha^2\beta^2) \text{ on } S,$$

where S denotes wing surface projected onto xy plane. Author shows how the solution can be set up by use of a fundamental solution (source). Because of effects of regions off the tips, in general, this leads to an integral equation, which can be inverted if $k = 0$, but if $k \neq 0$, it can be inverted only if wing has no subsonic trailing edge (in which case the region involved does not affect the wing).

The case of "homogeneous flow" is also treated, i.e., $\varphi_n = r^n f_n(\theta, \chi)$, where $r^2 = y^2 + z^2$, $\theta = \tan^{-1}(z/y)$, and χ denotes $x/\beta r$. If $2u$ denotes $x^2 - \beta^2 r^2$, a separation of variables is achieved, and the result is finally put into the form $\varphi_n = x^n F_{n+1/2}(2k^2u)H_n(\chi, \theta)$, where $F_{n+1/2}(2k^2u)$ is given by a relatively simple infinite series and $x^n H_n(\chi, \theta)$ is the solution for a steady-flow problem ($k = 0$). It is proposed to construct practical solutions by superposition.

As examples, oscillating flat plate of infinite span and oscillating delta wing are considered. Latter is characterized by $(\varphi_z)_{z=0} = \sum_{j=1}^{\infty} a_j x^{j-1} \exp\{-i\omega xV/\alpha^2\beta^2\}$ and the solution is obtained, in a rather complicated series form, by superposition of the solutions for homogeneous flows.

W. R. Sears, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 148, 318)

381. **Himmelskamp, H., Profile investigations on a rotating propeller** (in German), *Mitt. Max-Planck-Institut für Strömungsforschung,* no. 2, 38 pp., 1950.

Pressure distribution along one blade of a ducted two-blade propeller is measured at five span stations with 19 pressure taps each. Profile (Göttingen 625) and chord are constant along blade and nominal pitch is 1.06. Propeller is mounted upstream on same shaft with an axial blower and runs at a constant 2000 rpm. Air-flow velocity is varied by adjusting setting angle of the

12 blades of blower fan. In this way advance ratios from 0.124π to 0.459π can be realized. Provision is also made for a boundary-layer probe that permits measurement of velocity direction and magnitude at a few selected points up to a distance of 3.5 mm from propeller wall. Techniques of all these difficult measurements are discussed in detail.

Velocity distribution across jet is measured immediately fore and aft the propeller by a directionally sensitive impact tube. Values so determined are used to compute effective velocity distribution at propeller upon which final reduction of measured pressure distributions to section lift and drag coefficients are based. Reliability of these results is essentially limited by uncertainty inherent in determination of effective velocity, and in particular of its direction. It is, on the other hand, possible to determine the same coefficients from momentum transfer to fluid, and the latter is completely given by fore and aft velocity distributions alone. The two results, however, do not agree, and possible causes of discrepancy are discussed. For results obtained by momentum method, a correction seems feasible that makes initial portions of the section lift curves coincide with lift curve of an airfoil with same profile measured in wind tunnel.

It is remarkable that the maximum lift coefficient increases from tip section toward hub section, where it reaches the value 3.2. This postponement of separation to very high angles of incidence can be understood by considering the radial velocity component of boundary layer flow, which has an over-all effect similar to removal of boundary layer by suction.

G. Kuerti, USA

382. Haines, A. B., A comparison of the measured and calculated twist along a propeller blade, *Aero. Res. Counc. Rep. Mem.* R. & M. 2360, 8 pp., 1950.

A second-thinned version of a de Havilland Spitfire I propeller, set at 15° at the 0.7 radius, was tested in the 24-ft tunnel at V/nD of 0.2 and 0.3, and at tip Mach number of 0.75. Measured twist agreed with calculations at 0.7 radius but was about 20% greater at 0.9 radius. Allowing for effects of bending deflections and assuming center of pressure to move forward with Mach number on tip sections thinner than 6% reduces discrepancy. Based on these data, the method of calculating propeller twist is claimed to be satisfactory for untested metal blades below shock stall. Experimental twist measurements may be used to calculate qualitatively the variation of center of pressure with Mach number; quantitative calculations are laborious and recommended for special cases only.

H. P. Liepman, USA

383. Owen, J. B. B., Some notes on the flapping motion of rotor blades, *Curr. Pap. aero. Res. Counc. London*, no. 1, 17 pp., 1950.

Author investigates stability criteria for flapping of rotor blades, using equations derived in appendix of paper. Effects of a controlled pitch change with flapping and influence of offset between inertial and flexural motion upon blade lift and aerodynamic moment are not considered. Since the procedure is not applied to an actual problem nor compared with experiments, it is not possible to evaluate importance of these omissions.

F. Everett Reed, USA

384. von Krzywoblocki, M. Z., Elementary propeller theories in compressible inviscid fluids, *Appl. Sci. Res.* **A 2**, no. 3, 205–224, 1950.

Existing propeller theories are valid for an incompressible inviscid fluids range. In paper, the known Glauert's propeller theories are transferred into compressible inviscid fluids region. All discontinuity phenomena are excluded from considerations. Four elementary theories are discussed: axial momentum, general momentum, improved general momentum, and vortex theory. Application of some approximate methods of solution, like Pistolesi's method, to compressible range is shown. In a first approximation, Betz's theory of propellers of highest efficiency may be applied to propellers moving in a compressible medium, with assumption that conditions existing in propeller's wake are invariable and that compressibility phenomena are restricted to close neighborhood of propeller with density in propeller's plane differing from that in the wake. There are indications, however, that Betz's theorem should be revised for compressible range since the Froude-Finsterwalder theorem, on which the previous one is based, breaks down for compressible fluids as Frankl has shown.

From author's summary by A. A. Regier, USA

385. Hammack, Jerome B., and Vogeley, A. W., Propeller flight investigation to determine the effects of blade loading, *Nat. adv. Comm. Aero. tech. Note* 2022, 38 pp., Jan. 1950.

This article reports results of further flight tests to ascertain influence of propeller power loading at high speed and under climbing conditions upon propeller efficiency.

F. Everett Reed, USA

386. Haines, A. B., Comparative tests on propellers with simulated ice and with de-icing overshoes in 24-ft. tunnel, *Aero. Res. Counc. Rep. Mem.*, R. & M. 2397, Dec. 1946, 11 pp., 1950.

It is shown that ice accretion on the blades produces an efficiency loss under typical operating conditions of about 14%, while the particular feed shoes tested give a permanent loss in cruising efficiency of about 4–5% and hence of 3–4% in available range for a given pay load.

From author's summary

387. Kondo, Kazuo, On the vortex theory and the acceleration potential (in Japanese), *J. Soc. appl. Mech. Japan* 2, 27–29, 1949.

Theories of wing and propeller are divided into two types: Vortex theory, and theory based on acceleration potential. It is shown that both theories are identical and give same results.

Courtesy of Mathematical Reviews T. Okamoto, Japan

388. Hirsch, René, Determination and calculation of optimum airplane propellers, simple and coaxial (in French), *Publ. Sci. Tech. Ministère Air*, Paris, I, II, 220, 225, vi + 192, ii + 129, 1948, 1949.

Exhaustive treatise on theory of propellers, begun prior to war, features attempts to account for compressibility effects, treatment of dual-rotating coaxial propellers, and consideration of structural and aerodynamic factors together in optimum-propeller design. Reference is made in a number of instances to wartime German results, especially in connection with compressibility effects on airfoils. In calculating induced velocities, author employs an approximate system made up of discrete helical trailing vortexes. For dual-rotating propellers he estimates the periodic variation of blade circulation, including a rough account of unsteady-lift effects, and decides that it can be neglected. Supposing that optimum circulation distribution has been determined, author proposes to design propeller blades so as to minimize efficiency loss due to drag. Drag increases torque and reduces thrust. Problem becomes one of optimizing the function lC_x, where l is blade chord and C_x local drag coefficient, which depends on skin friction, profile curvature, lift coefficient, thickness ratio, Reynolds number, and Mach number, and at the same time meeting requirements of structural strength. Last two chapters are devoted to problems of propeller vibration and dynamics of

propellers with automatic blade-angle regulation. In these cases, unsteady-lift effects are neglected. W. R. Sears, USA

389. **Manson, S. S., Meyer, A. J., Calvert, H. F., and Hanson, M. P., Factors affecting vibration of axial-flow compressor blades,** *Soc. exp. Stress Anal.* **7,** 1–15, 1950.

Measurements were made by means of wire-resistance strain gages of blade vibrations in an experimental 10-stage axial-flow compressor during engine operation. Information is given regarding: (1) Effect of centrifugal force on natural frequency of blades; (2) common modes of vibration present and orders of excitation producing them; (3) effect of disturbances in air flow originating in inlet passage on vibration in each of the 10 stages; (4) use of loosely mounted blades as a means of vibration suppression; and (5) importance of aerodynamic damping in limiting vibration. R. C. Binder, USA

390. **Pearson, H., and Bowmer, T., Surging of axial compressors,** *Aero. Quart.* **1,** 195–210, Nov. 1949.

Elementary theories in regard to causes and occurrence of surging in axial-flow compressors are reviewed in light of extensive experience on test stand and in practice. It is shown that analysis of stability of steady flow in compressors, results of which place the surge point well down on rising branch of pressure-volume characteristic, is in contradiction to practical experience according to which surging occurs close to highest pressure. Neither is the theory that organ-pipe effects in the discharge duct dominate surging tenable in practice. Authors believe that dynamic characteristics of individual stage must be accounted for in a logical explanation of phenomenon of surging. To establish relationships of dynamic stability, characteristics of compressor stages, such as capacity, inertance, and impedance are defined in analogy to corresponding electrical quantities. The multistage compressor, treated like a series of coupled electrical circuits, is investigated for both critical divergence and divergent oscillations.

Results of extensive calculations, carried out step-by-step for a typical multistage axial-flow compressor, predict surging very near point of maximum pressure for both the case of divergence and of oscillations. While both forms of instability are possible, tests seem to indicate that actual surging is of nature of critical divergence.

Although mutual interference between stages was neglected and dynamic stage characteristics were assumed the same as steady flow characteristics, thus admitting of linearization of problem, nevertheless, the theory offers a logical explanation of phenomenon of surge in axial-flow compressors and a basis for predicting occurrence of surge in good agreement with practical experience. J. R. Weske, USA

391. **Merchant, W., An approximate investigation of the off-design performance of a turbo-compressor stage,** *Proc. Instn. mech. Engrs.* **161,** no. 53, 227–232, 1949.

Paper deals with the simple problem of an isolated stage of a turbine or compressor, working in a long, parallel annulus with a perfect, incompressible fluid. If work done is not constant along a radius, then velocity, which is constant before the stage, must vary along a radius downstream of the stage. Thus, radial shift of streamlines must be present. Author shows, taking as an example a fluid streaming without swirl, how the above flow can be predicted. He assumes that effects of a stage can be represented by an ideal stage of negligible axial width; and that velocity at the stage is the mean of upstream and downstream velocities at same radius. For purposes of calculation, velocity changes after the stage are taken as small, so that, as in case of radial shifts, they can be first of all neglected or represented by a mean value. Result can then be corrected by iteration. According to the assumptions, there exists at smaller flow a boundary at which velocity behind the stage approaches zero. Author believes that sometimes the instability at small flow may depend on this fact. Using calculated velocity profile behind the stage, distribution of total head and total work done is found. Furthermore, an efficiency is calculated belonging to energy loss by damping down the velocity profile to a uniform profile by viscosity, but without wall friction. Albert Betz, Germany

392. **Sanders, John C., and Chapin, Edward C., Equilibrium operating performance of axial-flow turbojet engines by means of idealized analysis,** *Nat. adv. Comm. Aero. tech. Note* 1956, 40 pp., Oct. 1949.

Paper deals with method for predicting equilibrium-operating performance of axial-flow turbojet engines starting from simplified model processes for the components.

Equilibrium operation points are fixed by three equations: (1) Compressor air flow plus fuel flow equals gas flow through turbine; (2) gas flow through turbine equals flow through exhaust nozzle; and (3) compressor power equals turbine power.

Results are given in a number of graphs in which temperature parameter or thrust parameter is plotted against engine-speed parameter, containing ratio of compressor cross section and turbine cross section, for several values of nozzle area/turbine area and ram pressure ratio. The ratio of compressor tip speed to compressor inlet velocity is considered as a constant, a reasonable approximation for axial-flow compressors. Ram efficiency is kept constant and equal to 100%. If characteristics of turbine and compressor are known, performance of any axial-flow turbojet can easily be calculated.

Correlation of experimental data with theory seems good. In case of a variable exhaust nozzle area, theory permits determination of correct area for any flight condition. J. G. Slotboom, Holland

Flow and Flight Test Techniques

(See also Revs. 55, 317, 346, 368)

393. **Kovásznay, Leslie S., The hot-wire anemometer in supersonic flow,** *J. aero. Sci.* **17,** 9, 565–572, 584, Sept. 1950.

Author discusses development, theory, and tests of the hot-wire anemometer for measuring supersonic flow up to Mach numbers of 2.0. He finds that the heat loss is a function of Reynolds number for small temperature loadings, and that it departs from King's law for large temperature loadings. Author finds that if turbulence level is low and pressure fluctuations can be neglected, then velocity and density fluctuations can be evaluated and the correlation obtained between the two. G. N. Cox, USA

394. **Wingham, P. J., An application of strain gages to the measurement of normal force and moments in high-speed wind tunnels,** *Aero. Res. Coun. Rep. Mem.*, R & M 2316, 12 pp., 1950.

Model is mounted on a sting, which passes through its body without contact and is fixed solidly in its nose. Strain gages are attached to this sting within the body, so that they are shielded from air flow. Air loads on model cause sting to deflect as a cantilever about point where it is rigidly attached to tunnel at its downstream end; these deflections, as recorded by strain gages, can be converted to give magnitude and position of normal force acting on model. By putting all four arms of the Wheatstone bridge circuit in tunnel, inaccuracies arising from temperature differences are eliminated. Other advantages claimed for the

circuit are that it can measure $^1/_{250}$ lb.-in. under steady conditions, bridge requires no final balance, and long connecting wires are no longer part of bridge.

Defects of system for altering incidence of model are discussed, particularly those encountered at low incidences. Vibration of model was another drawback, and it is suggested that a thicker sting might reduce this; improved circuit is, in fact, sufficiently sensitive to allow for this thickening. Further work at the Royal Aircraft Establishment is described in an appendix.

Joseph Black, England

395. Matthews, Clarence W., The design, operation, and uses of the water channel as an instrument for the investigation of compressible-flow phenomena, *Nat. adv. Comm. Aero. tech. Note* 2008, 26 pp., Jan. 1950.

Author concludes that the hydraulic analogy can be used to demonstrate many two-dimensional compressible-flow phenomena. Its main drawback is the comparatively low value of Reynolds number obtainable in the water channel of moderate size. A channel permitting Reynolds number of 3,000,000 would have to be approximately 20 feet wide if water temperature were 100 F.

A. Petroff, USA

396. Todd, K. W., Some developments in instrumentation for air-flow analysis, *Proc. Instn. mech. Engrs.* **161,** 53, 213–226, 1949.

Evolution of research into flow conditions in confined spaces, such as model ducts or blade cascades under high subsonic or transonic air velocities, has necessitated development of specialized forms of instrumentation for both optical and physical analysis. In paper, some of these special instruments are described, and an example of their application to a compressor-cascade examination in a high-speed wind tunnel is included.

From author's summary

397. Mortensen, Tage A., An improved schlieren apparatus employing multiple slit-gratings, *Rev. sci. Instrum.* **21,** 3–6, Jan. 1950.

A schlieren system is described which does not require the customary formation of a secondary light source by focusing light from the primary source on a slit. Instead, a ladderlike grid is inserted into the light path immediately behind the condenser lens which otherwise would have been used for formation of the secondary source. In the plane, conjugate to that of this first grid with respect to the objective lens, a second grid is placed such that its rungs are located where, in the undisturbed case, the images of the first grid's interstices are formed. Refractive deflections of the light paths between first grid and objective lens will shift image of first grid from its undisturbed position, creating conventional schlieren effects, provided geometry and slit width are adjusted so that no displacements in excess of one slit width will arise. (If slit width is small enough so that displacements generally amount to a few multiples of this width, then the record obtained is one of alternately dark and light fringes which mark the loci of constant displacements in multiples of slit width.)

F. J. Weyl, USA

398. Guthmann, Durt, Temperature measurements on metallic fuses (in German), *Arch. tech. Messen,* no. 166, V2164–1, Nov. 1949.

399. Stringer, J., The metering of liquid fuel flow, *J. Iron Steel Inst.* **163,** part 2, 168–172, Oct. 1949.

Orifice meters used for measuring flow of viscous fluids, such as fuel oils, are affected by changes in viscosity and specific gravity. A nomogram, given in paper, facilitates correction for these inaccuracies; it is limited to square-edged or sharp-edged orifices in range of Reynolds numbers between 500 and 3000. Calculations apply to mass rate of fuel oil flow, viscosity of which extends over range between 350 and 1000 sec Redwood no. 1, at 100 F, and specific gravity between 0.92 and 0.97 at 60 F.

Effect of temperature is discussed and it is found that an error of approximately -1% is caused by a rise in oil temperature from 9 to 13 F, depending upon its viscosity. To insure consistency within $\pm 1\%$ of indicated flow, close control of the oil temperature is shown to be necessary. Paper gives procedure for calculating from nomogram required temperature to reduce error to zero; it is recommended to maintain oil temperature within ± 10 F of that temperature. The special orifices with constant coefficients over a wide range of Reynolds numbers are still subject to changes in specific gravity, and it is therefore suggested to keep the temperature such that density is within $\pm 2\%$ of calibration value.

G. A. T. Heyndrickx, Belgium

400. Hubbard, Philip G., Application of the electrical analogy in fluid mechanics research, *Rev. sci. Instrum.* **20,** 802–807, Nov. 1949.

A technique is described for accurate determination of velocity and pressure distribution of potential fluid motion along three-dimensional boundaries by means of electrical measurements upon an electrolytic bath having similar geometrical boundary as that of flow system under consideration. Obtained results are in good agreement with water-tunnel measurements.

Minglung Pei, USA

401. Linford, A., Flow measurement and meters, London, E. and F. N. Spon, Ltd., 1949, xx + 336 pp., 136 figs. £1.10s.

Book gives a general picture of art of metering flow of fluids. Flow meters are grouped under two classes: Meters for measuring flow through closed conduits (mechanical flow meters, differential pressure flow meters, shunt type flow meters, and variable aperture flow meters); and meters for measuring flow through open conduits (weir, Venturi flume, and meters for stream gaging). Last chapter of book considers subject of telemetering.

Albert Schlag, Belgium

402. Simmons, L. F. G., A shielded hot-wire anemometer for low speeds, *J. sci. Instrum.* **26,** 407–410, Dec. 1949.

Developed with a view of eliminating the major drawbacks precluding a more general use of hot-wire anemometers, this instrument consists of a nichrome heating wire and a nichrome-constantan thermocouple placed closely adjacent to each other in the bores of a twin-bore silica tube. This tube is $^1/_{32}$-in. diam and 1-in. long, the wires 0.008-in. diam, approximately.

Calibrations at low velocities up to 2 fps by whirling arm and wind-tunnel methods were in mutual agreement. Convective air currents were found to cause errors at air velocities below 0.25 fps; their effect may be eliminated, however, by orienting the tube vertically. Calibrations include effect of variation of air temperature at very low velocities. Pressure effects are nil. By careful control of heating current which is kept constant during use of instrument, reliable readings of mean velocity were obtained over long intervals. Instrument is insensitive to turbulence.

J. R. Weske, USA

403. Cope, W. F., Watson, G. G., Preliminary measurements of the boundary layer in the 11-in. supersonic wind tunnel, *Rep. Memo. aero. Res Counc. London,* no. 2304, 7 pp., 1946; published 1949.

Velocity and Mach number distributions in boundary layer of

the National Physical Laboratory 11-in. supersonic wind tunnel have been measured at Mach numbers of 1.5 and 2.4. The Reynolds number, based on distance from throat as representative length, was varied from $2^1/_4$ million to $7^1/_4$ million. Results show that boundary layer is supersonic to within a very short distance of wall, and that representation of distribution by either a logarithm or a power law becomes less accurate as Mach number increases. From author's summary by W. C. Nelson

404. Wuest, W., **Tubular spring pressure gage special problems** (in German), *Arch. tech. Messen* **156**, 86–88, Aug. 1948.

Thermodynamics

405. Curtiss, **Charles F., and Hirschfelder, Joseph O., The thermodynamics of flow systems,** *J. chem. Phys.* **18**, 171–173, 1950.

In a treatment of matter in gross mechanical equilibrium, one can define entropy and then, through a statistical mechanical analysis, prove the second law. Authors have done the same for nonequilibrium flow systems. Defining entropy in exactly the same manner as the static case, they prove by use of nonequilibrium statistical mechanics (kinetic theory) that increase in entropy due to irreversible processes involving heat conduction, viscosity, diffusion, and chemical reactions is positive or zero. This is stronger than the second law of thermodynamics, which only requires that this be true of the entire system. If the mechanism of energy transfer is radiation, the stronger statement would no longer apply. H. S. Tsien, USA

406. Harle, G. A., **Measurement of the ratio of the principal specific heats of a gas,** *Nature* **165**, 4185, p. 74, Jan. 1950.

Method involves observation, on a U-manometer containing mercury, of the growth of pressure with time in an initially evacuated chamber into which atmospheric air is effusing through an orifice. Ed.

407. Potter, **Robert L., Thermodynamic properties of oxygen fluoride and chlorine fluoride from spectroscopic data,** *J. chem. Phys.* 17, 957–959, Oct. 1949.

Spectroscopic and electron diffraction data have been combined to make calculations of free energy, entropy, and heat capacity of OF_2 to the rigid rotator-harmonic oscillator approximation between 298.16 and 1500 K. The spectrum of ClF is known well enough to calculate these properties to the next approximation from 298.16 to 2000 K. Thermal data have been combined with these results to give complete thermodynamic properties of these two fluorine compounds. It is pointed out that the accepted dissociation energy of F_2 is too high, and calculations show that it must be close to 1.5 ev per molecule.

W. M. Rohsenow, USA

408. Meyers, **Cyril H., Table and Mollier chart for ammonia below −60 F,** *U. S. Dept. Commerce. Nat. Bur. Stands. Circ.* C 472, 3 pp., Nov. 1948.

Paper contains a table of thermodynamic properties of saturated ammonia vapor and liquid below −60 F, and a Mollier chart for saturated and for super-heated ammonia vapor covering the pressure range 0.8 to 5 psi. These data are based on same equations and are expressed in same units as those used in *Nat. Bur. Stands. Circ.* C142. Table and Mollier chart are extensions of those given in C142 to lower temperatures and pressures, terminating at triple point of ammonia (−107.86 F, 0.878 psi). From author's summary

409. **Popoff, Kyrille, Nearnst's fundamental theorem and the theory of real gases** (in French), *Proc. seventh int. Congr. appl. Mech.* **3**, 29–39, 1948.

The author proposes a new equation of state for gases, which satisfies the third law of thermodynamics, but does not fit any actual data on gas properties. Serge Gratch, USA

410. **Benson, Sidney W., and Gerjuoy, Edward, The tensile strengths of liquids. I. Thermodynamic considerations,** *J. chem. Phys.* **17**, 914–918, Oct. 1949.

An investigation is made of thermodynamic stability of liquids under tension. Comparisons of Helmholtz free energies show that small samples of liquid under tension are unstable with respect to two-phase liquid-vapor equilibrium at the same volume and temperature.

Defining tensile strength as negative pressure at which $(\partial P/\partial V)_T = 0$ for the liquid, it is possible to calculate "maximum" possible tensile strengths from equations of state. Van der Waals and Berthelot equations in reduced form can be adapted to such explicit calculation and give values much lower and therefore much closer to experimental values, than previous calculations. For nonassociated liquids at 27 C, the van der Waals maximum tensile strengths are about 150 atm, whereas the Berthelot values are about 750 atm. Temperature dependence for the two equations is about 1 and 6 atm °C, respectively.

Comparisons with observation of the calculated reduced liquid volume, energy of vaporization, coefficient of thermal expansion, and bulk compressibility support the use of the van der Waals and Berthelot equations in computing liquid properties.

W. M. Rohsenow, USA

411. **Hulburt, Hugh M., and Hirschfelder, Joseph O., On the non-equilibrium theory of absolute reaction rates,** *J. chem. Phys.* **17**, 964–968, Oct. 1949.

A chemical reaction is here represented by motion of a mass point in the configuration space of the reacting complex. Hydrodynamic equations are derived for mean motion of the ensemble of such point masses. Under assumption of no collisions between complexes, these equations are shown to have a form quite similar to continuity and momentum equations for a gas moving adiabatically.

One-dimensional equations are discussed in some detail, with emphasis on velocity of sound and on shock waves, including a term in former representing potential energy. It is pointed out that deviations from equilibrium density distribution, due to reflection by the potential energy surface can only accentuate the normal (i.e., positive) density gradient across shock waves. Hence no shock wave can persist between initial and activated states, but may develop after the flow passes the activated state into a region of increasing density.

Primary purpose of paper is to show that no changes in energy distribution among the several degrees of freedom of a reaction complex can occur before the activated state is reached unless, as rarely occurs, the complex suffers a collision with a third body. Hence, the Eyring theory of absolute reaction rates might be applicable to nonequilibrium processes.

In connection with this subject, mention may be made of work (not referred to in paper) of Bethe and Teller ["Deviations from thermal equilibrium in shock waves," BRL Rept. no. X117, Aberdeen Proving Grounds, Md.], showing, for example, that equilibrium for the vibrational modes is obtained less rapidly than for the translational and rotational degrees of freedom.

Morris Morduchow, USA

412. King, R. O., and Durand, E. J., The oxidation, ignition, and detonation of fuel vapors and gases. X. The cause of pre-ignition (surface ignition) undoped liquid fuel, *Canad. J. Res. Sec. F.* **27,** 337–346, Sept. 1949.

Theory is advanced that pre-ignition during compression in Otto cycle engines is due to partial oxidation of the fuel to aldehydes, a relatively small concentration of these inflammables being sufficient to cause ignition of the charge by hot surfaces normally present in high duty engines. CFR engine experiments are reported which are interpreted to substantiate this theory. Authors conclude that pre-ignition occurs on hot surfaces and not by detonation originating in homogeneous end gas. Effects of observed phenomena on engine operation and life are discussed.

J. Howard Childs, USA

413. King, R. O., and Durand, E. J., The oxidation, ignition, and detonation of fuel vapors and gases, XI. The loss of power coincident with the antiknock action of the carbonyls of iron and nickel in rich mixtures, *Canad. J. Res. Sec. F.* **27,** 347–358, Sept. 1949.

CFR engine experiments show that increase in usable compression ratio with iron or nickel carbonyl doped fuel is not accompanied by commensurate increase in power. Using doped fuel containing 0.5 cc per liter of iron carbonyl, the power was less than that obtained with a nonknocking fuel at the same compression ratio by 2% for maximum power mixture, and by over 22% for extremely rich mixtures. Authors interpret this power loss to be caused by appreciable oxidation of the fuel prior to completion of compression when doped fuel is used. Such a conclusion supports the theory advanced in part III.

J. Howard Childs, USA

414. Hirschfelder, J. O., and Curtiss, C. F., The theory of flame propagation, *J. chem. Phys.* **17,** 1076–1081, Nov. 1949.

This very complete formulation of the theory of flame structure based on kinetic theory derivation considers variation of material properties within flame and allows for chemical reaction and diffusion of N chemical species, together with energy transport. One-dimensional (stationary) flame is treated in detail and leads to nonlinear eigenvalue problems which are of considerable difficulty for all but simplest reactions. Discussion of specific flame systems has been given by J. Corner [*Proc. roy. Soc.* **197,** p. 90, 1949; **198,** p. 388, 1950] and by Hirschfelder, et al ["Third symposium on combustion, flame and explosion phenomena," 121–140, Williams and Wilkins, 1949].

Bruce L. Hicks, USA

415. Bechert, C., On the theory of flame propagation with application to combustion of ozone (in French), *Portugaliae Phys.* **3,** 29–90, 1949.

Theoretical determination of normal flame velocity u_l from the basic principles of hydrodynamics and chemical kinetics was first made by Lewis and von Elbe [*J. chem. Phys.* **2,** 537–546, 1934] for the ozone decomposition reaction. To make computation tractable, authors were forced to use an important simplifying assumption. A complete but very complex formulation of this problem was given by Hirschfelder and Curtiss [*J. chem. Phys.* **17,** 1076–1081, 1949]. Present paper departs from both these treatments in that no attempt is made to give a fundamental theory; only semi-empirical or phenomenological analysis is given. This is achieved through introduction of concept of a single "over-all reaction" with an activation energy E. Molar fraction of combustible is denoted by $1/(1 + m^*)$, ratio of molecular weights of combustible and product by μ. Then result of calculation shows that

$$u_l = \frac{(2A_1)^{1/2}(m^* + 1)(m^* + \mu)^{-1}e^{-\tau_r/2}}{\tau_l\tau_r(1 - \tau_r/\tau_l)^{1/2}} \qquad [1]$$

where the τ's are defined as $\tau \equiv E/kT$, k Boltzmann constant, and subscripts l and r refer to unburned and burned gas; A_1 is related to properties of reacting molecules. Author considers A_1 and E as constants to be determined from, say, two experimental measurements on u_l. Then (1) can be used to predict u_l at other conditions. Author obtains good correlation with experiments on decomposition of ozone by Lewis and von Elbe [*J. chem. Phys.* **2,** 283–290, 1934].

For detailed calculation, author uses three equations: Continuity, momentum, and energy equation. Viscous stress is neglected in the momentum equation and diffusion is accounted for by an appropriate modification of E. Introducing boundary condition that $(d^2T/dx^2)_l = 0$, x space coordinate, author shows that $(dT/dx)_l$ is very small, i.e., space variations of temperature and other physical variables in unburned gas are very small. Equations are then reduced to a single first-order differential equation with independent variable τ, and parameter u_l. Since there are two end conditions at unburned side and burned side to be satisfied, parameter u_l can be determined. Actual solution is done by first assuming a proper approximate solution which satisfies end conditions and then "forcing" this solution to fit the differential equation at some point. Choice of fitting point does not influence the result greatly and [1] is the averaged result.

H. S. Tsien, USA

416. Schweikert, G., On the theory of gas pressure against a moving wall (in German), *Z. angew. Math. Mech.* **29,** 289–300, Oct. 1949.

Purpose of paper, an extension of another paper by same author, is to develop a theory of gas pressure against a moving wall. Theory is based on kinetic theory of gases. In general, there are three fundamentally different items which exert an influence on value of pressure of a gas against a moving wall, namely: (1) Change in impulse (which is exerted by molecules on wall) due to motion of wall; (2) change in velocity component perpendicular to wall of molecules after reflection from moving wall; this change caused by motion of wall is the mechanical reason for change in temperature of gas at adiabatic conditions, which are assumed to be valid; and (3) change in density of gas close to moving wall due to change in volume caused by motion. In present theory third item is completely neglected and the second only partially taken into account; e.g., author considers influence of the change in velocity component (after reflection) on value of pressure and not on value of temperature. Presumption is that density and temperature of gas immediately at moving wall are known. One more item must be taken into account, namely, law defining relation between magnitude of molecular velocity component perpendicular to wall and number of molecules possessing that velocity component. Since there is no such rigorous law derived as yet, Maxwell's distribution law is used, although it causes a certain idealization of actual gas conditions. Under those assumptions author derives formula for pressure of gas in integral form. Applied mathematics are simple and do not go beyond Gaussian error integral. All integrals are calculated numerically and several tables give values up to fifth and even sixth decimal place. Formula obtained for gas pressure shows that, while pressure of an ideal gas drops asymptotically to zero with increasing wall velocity for a velocity infinitely great, pressure of a real gas declines to zero at finite wall velocities, depending upon deviation from state of ideal gas. Author applies obtained formula to derivation of value of drag of a moving body for two extreme velocity cases (large and small). Final formulas differ from those known in ordinary aerodynamics. No comparison with tests is

given and it is not possible to judge how well presented theory is applicable, particularly in range of hypersonics.

M. Z. Krzywoblocki, USA

417. **Guggenheim, E. K., Statistical basis of thermodynamics,** *Research London* **2,** 450–454, Oct. 1949.

This survey article summarizes interrelationship between statistical mechanics and thermodynamics. Main question discussed is how, and in what form, laws of thermodynamics follow directly from a single assumed statistical principle of a general nature. Justification of this principle on basis of quantum theory is not attempted, but rather the a priori postulatory basis of statistics is taken to be (following Darwin and Fowler) that measurable properties of a closed system having a prescribed energy E are correctly obtained by averaging over-all accessible independent quantum states attaching equal weights to all such states. It is shown that, if the entropy $S(E)$ of a closed system is defined as $S(E) = \text{const} \log \Omega(E)$ and the Helmholtz free energy $F(T)$ as $F(T) = -kT \log Q(T)$, where Ω is number of accessible independent quantum states and $Q(T)$ the partition function, usual thermodynamic relations between total energy, entropy, and free energy for macroscopic systems follow directly.

Novel features of articles' presentation consist in a discussion of the thermodynamic properties of microscopic systems, i.e., those having a small number of degrees of freedom, and an unequivocal physical interpretation of entropy as a measure of "spread" or accessibility without necessity of introducing concept of less probable or more probable states.

L. M. Grossman, USA

418. **Livingston, H. K., The cross-sectional areas of molecules adsorbed on solid surfaces,** *J. Colloid Sci.* **4,** 5, 447–458, Oct. 1949.

According to Brunauer-Emmett-Teller theory, it is possible to calculate cross-sectional areas of adsorbed molecules based on a standard value for nitrogen. Cross-sectional areas for 23 different molecules are calculated with this method. Values are consistent with areas predicted on basis of liquid densities and two-dimensional van der Waals constants. Adsorption isotherms according to Jura-Harkins and others are also discussed. It is found that these latter isotherms are less suitable for cross-sectional area calculations than the one according to Brunauer-Emmett-Teller.

E. Haenni, Switzerland

419. **Van Krevelen, D. V., Kinetics of interphase rate processes,** *Ingenieur' s-Gravenhage* **61:** 42, 34–37, Oct. 1949; 44, 41–44, Nov. 1949.

420. **Wall, Frederick T., A theory of vapor pressures of liquids based on van der Waals' equation of state,** *J. chem. Phys.* 16, 5, 508–512, May 1948; **Wall, Frederick T., and Stent, Gunther S., A theory of vapor pressures of liquids based on van der Waals' equation of state, II. Binary mixtures,** *J. chem. Phys.* 17, 11, 1112–1116, Nov. 1949.

An expression is derived for vapor pressure of a liquid assuming validity of van der Waals' equation of state. Theory involves use of an appropriate mathematical expression for volume of a van der Waals liquid and does not require recourse to any empirical relationships. Vapor pressure equation so obtained can be put into a reduced form, which is of a universal nature. The laws of Guldberg-Guye and of Trouton are shown to be compatible with the theory with constants of correct order of magnitude.

An expression for vapor pressures of binary liquid systems obeying van der Waals' equation of state is developed by generalizing a recent theory of vapor pressures of pure liquids. Resulting partial vapor pressure equations involve composition and temperature, and contain no empirical quantities other than van der Waals' constants, including an interspecies interaction term. Conditions under which Raoult's law holds for these systems are formulated, together with conditions for which positive or negative deviations occur. Compositions at which azeotropes form are predicted, and the partial miscibility of binary liquid systems is discussed. A number of examples are calculated to illustrate the theory.

From authors' summary by Joseph Kaye, USA

421. **Gorter, C. J., and Frederikse, H. P. R., A few remarks on physical adsorption,** *Physica Hague* **15,** 891–896, Oct. 1949.

Phenomenon of physical absorption may be treated by theories which are either thermodynamic or kinetic in nature. The kinetic approach is widely used and gives a valuable first picture, but paper develops an improved thermodynamic theory which is considered to be superior. A method of calculating entropies of absorption is given. Superiority of new treatment is supported by statistical arguments and experimental data.

C. W. Smith, USA

422. **De Boer, J., The caloric and thermal equation of state in quantum statistical mechanics,** *Physica Hague* **15,** 843–848, Oct. 1949.

Born and Green have suggested [*Proc. roy. Soc. A* **191,** 168, 1947] that, in connection with the equation of state, pressure obtained by statistical mechanics and dynamics is different from that obtained by partition-function method of statistical thermodynamics. In paper the two expressions for equation of state are developed in forms of probability densities and shown to be identical.

Keith J. Laidler, USA

423. **Green, H. S., The equation of state in quantized kinetic theory and quantum statistical mechanics,** *Physica Hague* **15,** 882–890, Oct. 1949.

Paper presents a rigorous derivation of equation of state of a fluid on the basis of quantum statistical mechanics. In a previous paper, de Boer [*Physica Hague* **15,** p. 843, 1949] obtained the equation of state by application of the virial theorem, and pressure so defined may be called *kinetic* pressure; that defined in terms of rate of change of free energy with variation of volume, and evaluated in present paper, is *thermodynamic* pressure. At high temperatures the two pressures are identical, but it is here shown that at low temperatures the pressures are not the same. This result implies that work done in adjusting a difference in pressure need not appear as macroscopic kinetic energy, but may be converted reversibly into thermal energy. It is suggested that this result explains, in part, the abnormal properties of liquid He II.

Keith J. Laidler, USA

424. **King, R. O., Durand, Edwin J., and Morrison, J. Alex., The oxidation, ignition, and detonation of fuel vapors and gases. XII. The high compression ratio Otto cycle gas engine and the adverse effect of high jacket temperatures on thermal efficiency,** *Canad. J. Res. Sec.* **F 27,** 435–449, Nov. 1949.

Town gas was used as fuel for CFR engine during a series of engine trials run at a compression ratio of 10:1 and at jacket temperatures of 140, 212, 302, and 351 F. Mixture strength at each jacket temperature was varied from weakest to richest on which engine would run steadily. Object of trials was (1) to determine effect of jacket temperature on thermal efficiency, and (2) to compare performance of an Otto cycle engine run at 10:1 compression ratio with that of a compression ignition (Diesel) oil engine. Results of trials show that thermal efficiency decreases as jacket

temperature is raised, i.e., the decrease more than offsets gain due to decrease of heat loss during combustion and expansion. It is indicated by character of results and by experiments described earlier that observed loss of efficiency is due to flameless surface combustion of hydrogen to form steam during compression.

Performance of the CFR engine running on town gas at 10:1 compression ratio as compared with that of a compression ignition oil engine running at 12:1 was superior in respect to maximum power (IMEP) developed and range of quality control and not greatly inferior in respect to thermal efficiency.

W. M. Rohsenow, USA

425. Meixner, Josef, Thermodynamics and relaxation (in German), *Z. Naturforsch.* **A 4a,** 8, 594–600, Nov. 1949.

Knowledge of relaxation phenomena, and especially of relaxation times, is of great help in study of atomic processes and properties. Author develops a theory of relaxation which can be used to calculate relaxation times. Theory applies to homogeneous systems in which an arbitrary reaction occurs (the term "reaction" including ordinary chemical reactions, dissociation, association, ionization, etc.), and is based on fundamental thermodynamic relations. Formulas are derived for both isothermal and adiabatic relaxation at constant volume, and it is demonstrated that the relaxation time is always less for adiabatic than for isothermal relaxation at constant density. Effect of a periodic external disturbing influence, such as an electric or magnetic field, on relaxation time is briefly considered. In general, the theory is supplementary to kinetic theory developed by Kronig in 1938, but is of somewhat broader application.

C. W. Smith, USA

Heat and Mass Transfer

⚘426. Kern, Donald Q., Process heat transfer, New York McGraw-Hill Book Co., Inc., 871 pp., 1950. $8.

Author states that "it is the object of this text to provide fundamental instructions in heat transfer while employing the methods and language of industry." First five chapters are devoted to an introduction to fundamental principles consisting of I, Process Heat Transfer; II, Conduction; III, Convection; IV, Radiation; and V, Temperature. These follow established treatments although, particularly with reference to convection, the latest generally available theories of Boelter, Martinelli and others are not mentioned, nor are numerical methods of solutions of conduction problems given sufficient emphasis. Remainder of text, the largest part, consists of specific applications with many fully worked-out examples, illustrating the principles of analysis clearly. A group of chapters pertain primarily to forced and free convection heat exchangers: VI, Counterflow; double-pipe exchangers; VII, 1–2 Parallel-counterflow, shell-and-tube exchangers; VIII, Flow arrangements for increased heat recovery; IX, Gases; X, Streamline flow and free convection; XI, Calculations for process conditions. The field of condensation, evaporation and boiling processes is next treated: XII, Condensation of single vapors; XIII, Condensation of mixed vapors; XIV, Evaporation; XV, Vaporizers, evaporators, and reboilers; XVI, Extended surfaces; XVII, Direct-contact transfer—cooling towers. These chapters assemble conveniently and clearly much information not available in such form elsewhere. A final group of miscellaneous topics completes the book: XVIII, Batch and unsteady state processes; XIX, Furnace calculations; XX, Additional applications; XXI, The control of temperature and related process variables. Book will be difficult to use as a text but will be of real value to personnel in industry who will find full treatment of many relevant problems.

N. A. Hall, USA

427. Lidyaev, S. F., On the representation of a solution of the equation of heat conduction in the form of a Poisson integral (in Russian), *Ucenye Zapiski Moskov. Gos. Univ.* **135,** *Matematika,* **II,** 86–109, 1948.

Concerning the Poisson integral, $p(x, t) = \frac{1}{2}\pi^{-1/2} {}_{-\infty}\int^{\infty} t^{-1/2} e^{(x-\xi)^2/(4t)}\phi(\xi)d\xi$, author first proves that if $p(x, t)$ converges for x_0, t_0, (A) then for some positive numbers, c, k, $e^{-cx^2}|p(x, t)| < kt^{-1/2}$ for $0 \leqq t \leqq t_0$; (B) the integral ${}_{-\infty}\int^{\infty} e^{-cx^2}p(x, t)dx$ converges uniformly in t. Then he shows that if $u(x, t)$ is a positive parabolic function (solution of $\partial^2 u/\partial x^2 = \partial u/\partial t$) such that $e^{-cx^2}|u(x, t)| < kt^{-1/2}$ and for which ${}^{-\infty}\int e^{-cx^2}u(x, t)dx$ converges uniformly in t, then $u(x, t)$ can be written as a Poisson-Stieltjes integral $u(x, t) = \frac{1}{2}\pi^{-1/2} {}_{-\infty}\int^{\infty} t^{-1/2}e^{-(x-\xi)^2/(4t)}d\phi(e)$, where the set function $\phi(e)$ is of bounded variation. Finally, if $u(x, t)$ is a positive parabolic function such that $\max_{0 \leqq t \leqq t_0} |u(x, t)|e^{-cx^2} < kt^{-1/2}$ and ${}_{-\infty}\int^{\infty} e^{-cx^2}u(x, t)dx$ converges uniformly t, and, further, if $F(e, \xi) = \int_e u(x, \xi)dx$ has the property of uniform absolute continuity, then $u(x, t)$ is expressible in the form of a Poisson-Lebesgue integral for all t in the interval $0 < t < t' - \xi$. Proofs of these theorems and lemmas leading up to them are given in considerable detail.

Courtesy of Mathematical Reviews R. E. Gaskell, USA

428. Bononcini, Vittorio E., A problem of heat propagation (in Italian), *Atti Sem. Mat. Fis. Univ. Modena* **3,** 142–161, 1949.

Let S be the rectangular region $0 < x < L$, $0 < y < l$, and let C be the boundary of S. Author uses method of separation of variables to solve following two boundary-value problems. In problem one, u is a solution in S of the equation

$$Ku_t = u_{xx} + u_{yy} + \lambda - bu \tag{1}$$

(λ, b constants), with $K = 0$, and $u = 0$ on C. In problem two, u is a solution of (1) in S for $t > 0$ and $K = 1$, meeting the boundary conditions $u(x, y, t) = 0$, $t = 0$, (x, y) in $S + C$; $u(x, y, t) = 0$, $t > 0$, (x, y) in C.

Courtesy of Mathematical Reviews F. G. Dressel, USA

429. Laasonen, Penti, On a method to solve the heat conduction equation (in German), *Acta Math.* **81,** 309–317, 1949.

Author solves the parabolic equation $\kappa^2 u_{xx} - u_t + q(x, t) = 0$ $(0 \leqq x \leqq L, 0 \leqq t \leqq T)$ subject to boundary conditions $u(x, 0) = f_0(x)$, $u(0, t) = f_1(t)$, $u(L, t) = f_2(t)$,where q_{xx}, q_t, $f_0^{(4)}$, f_1'', f_2'' are assumed to exist. He finds u by means of a modification of method applied by R. Courant, K. Friedrichs, and H. Lewy to the homogeneous heat equation on an infinite interval [*Math. Ann.* **100,** 32–74, 1928], in which u is represented as limit of solutions u_n of difference equations belonging to a rectangular lattice. Using a different form of the difference quotient, author establishes convergence of a suitable subsequence of the u_n towards a solution u of original problem, as the sides h and k of the fundamental rectangle of lattice tend to zero in any manner. In contrast to corresponding situation for hyperbolic equations no inequalities between h and k are needed here to assure convergence.

Courtesy of Mathematical Reviews F. John, USA

430. Landau, H. G., Heat conduction in a melting solid, *Quart. appl. Math.* **8,** 81–94, 1950.

Heat at the rate $H(t)$ per unit area is supplied at the moving face $x = s(t)$ of an infinite plane-parallel slab at initial temperature $T_0(x)$, the right face $x = a$ being insulated. The solid melts

at temperature T_m, and, in contrast to other similar problems, melted material is removed. After a discussion of the general problem, it is specialized by setting $H(t) = H_0$, $T_0(x) = T_0$, both constant, and letting $a \rightarrow \infty$. The specialized problem is simplified by elimination of moving boundary through use of variable $z - s(t)$, which represents distance from face of melting solid. New statement of problem is especially convenient because it involves only the single parameter $m = \pi^{1/2}c(T_m - T_0)/2L$, where L is heat of fusion, c specific heat. The steady-state solution (case of uniform progress of melting face) is given, as is solution for $m = 0$. Through numerical integration, $s(t)$ is found for $m = 0.2$, 1, 2, 5, 10 and ∞.

Courtesy of Mathematical Reviews R. E. Gaskell, USA

431. **Lyubov, B. Ya., Calculation of the rate of hardening of a metallic ingot** (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **68**, 847–850, 1949.

Solidification (crystallization) of a slab of material of uniform thickness and with insulated edges is considered, with special attention given to progress of front of crystallization into the slab. Material is first considered liquid with its temperature at freezing point. A boundary-value problem is then set up, involving heat equation, variable surface temperature, temperature at moving front of crystallization $y(\tau)$ and a relation expressing heat balance at $z = y(\tau)$. After a change of variables, $\xi = 1 - z/y(\tau)$, a solution of the form $V(\xi, \tau) = \sum_{n=0}^{\infty} a_n(\tau)\xi^n$ is assumed. A recursion formula for $a_n(\tau)$ is then found, after which use of surface condition $V(1, \tau) = f(\tau)$, and some manipulation of series, gives $y(\tau)$ as a series involving $f(\tau)$. Proof of convergence of resulting series is left to be considered for each special function $f(\tau)$. Result for $f(\tau)$ = constant is expressed in closed form. [Sign in equation (4a) is in error, but a second error made in transforming to (4b) corrects it.]

Courtesy of Mathematical Reviews R. E. Gaskell, USA

432. **Kreith, Frank, and Summerfield, Martin, Heat transfer to water at high flux densities with and without surface boiling,** *Trans. Amer. Soc. mech. Engrs.* **71**, 805–815, Oct. 1949.

An experimental study of heat transfer from a stainless-steel tube to water flowing through it was made for rates of heat flow up to 3 Btu/(sq. in.) (sec) and for rates of water flow up to 5.4 lb/(sq. in.) (sec). For heat transfer without surface boiling, it is concluded that Colburn equation $N_{uf} = 0.023R_{ef}^{0.8}P_{rf}^{1/3}$, where N_{uf} is Nusselt number, R_{ef} Reynolds number, P_{rf} Prandtl number, all based on arithmetic-mean temperature conditions, correlates experimental data. At equal Reynolds numbers, friction coefficient for flow with heat transfer is equal to friction coefficient for flow without heat transfer multiplied by the 0.13 power of ratio of viscosity of water at wall temperature to that at bulk temperature. Reynolds number of these tests varied between approximately 50,000 and 350,000 and heat flux varied between about 0.5 and 2.0 Btu/(sq. in.) (sec).

When surface boiling occurs, it is found that a large increase in heat flux occurs without a large increase in surface temperature, and that surface temperature required for transfer of a given amount of heat from tube to liquid depends mainly on pressure of liquid. Difference between surface temperature and boiling temperature of liquid, stated to be an important variable in heat transfer with surface boiling, varied inversely as absolute pressure raised to $^3/_4$ power for a fixed heat flux at a water velocity of 7 ft/sec. Influence of velocity of water in pipe is small compared with that of pressure. Amount of heat water could absorb at a given pressure and velocity is limited by an instability of flow.

Neal Tetervin, USA

433. **Harrison, W. B., Heat transfer to liquid metals flowing in asymmetrically heated channels,** *Trans. Amer. Soc. mech. Engrs.* **71**, 797–803, Oct. 1949.

Heat transfer to a fluid in turbulent flow between two plane parallel walls is discussed for case of an asymmetrically heated system in which entire heat flux is transferred at one of the walls and no heat passes through the other. A few remarks are made regarding experiments which might yield information on variation of eddy diffusivity near center line of a channel, and a method is presented for predicting heat transfer in asymmetrical heat-transfer systems. Work is an extension of work of R. C. Martinelli ("Heat transfer to molten metals," title source, **69**, 947–959, 1947). A discussion is given by Martinelli in which corrections to above reference are given, applicable to heat transfer in molten metals. Unit thermal conductances are lower in case of asymmetry. At low Peclet numbers, ratio of conductances for symmetrical and asymmetrical heating is one half.

Myron Tribus, USA

434. **De Haas van Dorsser, A. H., Leniger, H. A., van Meel, D. A., and Welling, W. A., Preliminary investigation on the influence of the degree of air turbulence on the rate of evaporation from a free water surface into an air stream,** *Ingenieur 's-Gravenhage* **61**, Oct. 7, 25–30; Oct. 21, 33–34, 1949.

Authors have measured rate of evaporation from a wet piece of blotting paper into an air stream at different velocities ($4.10^4 < Re < 2.10^5$). Degree of turbulence $\tau = 100\sqrt{v^2}/V$ was varied with aid of several grids from 0.3 to 13%. Flow distribution being considerably disturbed by turbulence generators, velocity immediately above test surface served as a basis for comparison.

Increasing degree of turbulence from 0.3 to 5% resulted in an increase of W, number of grams of water evaporated per sq m per hr per deg of wet bulb depression at room temperature. A further increase of τ did not influence W markedly. Accuracy was not sufficient to decide whether slope of the W vs. Re-curves was changed. Results are compared with measurements of Coming, Clapp and Taylor (cylinders normal to an air stream).

In order to determine τ, fluctuations v of absolute value of air velocity have been measured with aid of a method due to Betchov and Kuiper, using a hot-wire anemometer and compensating for its lag in phase and amplitude by means of an adjustable electrical impedance. To obtain an impression of structure of most coarse type of turbulence ($\tau = 13\%$) spectral distribution of v was measured. It showed a maximum at 35 cps; greater part of energy of turbulence belongs to low frequency components.

B. H. Schultz, Holland

435. **Laidler, K. J., and Shuler, K. E., The kinetics of membrane processes. I. The mechanism and the kinetic laws for diffusion through membranes, II. Theoretical pressure-time relationships for permeable membranes,** *J. chem. Phys.* **17**, 851–860, Oct. 1949.

In part I, equations for diffusion of components of a more concentrated binary solution to a less concentrated one through a membrane are discussed. Activities are utilized as the flow potentials rather than concentrations. This leads to usual mosotic pressure expression for case of zero flow rate. In part II, applications of equations to semipermeable membranes are given. Expression for rate of rise or fall of fluid in a capillary attached to a semipermeable membrane is presented. M. Tribus, USA

436. **Gutman, L. N., On laminar thermal convection above a stationary heat source** (in Russian), *Prikl. Mat. Mekh.* **13**, 435–444, 1949.

On the basis of equations of free convective laminar flow, the

flow pattern established by a stationary heat source is considered. It is shown that radial and vertical velocity components and temperature are given respectively by: $u = 2\mu r^{-1}(\eta F' - F)$, $w = (\lambda Q/\mu)^{1/2}F'$, $\theta = Q(\mu z)^{-1}F''$ where μ is viscosity, λ coefficient of thermal expansion of air, Q strength of heat source, and F solution of $\eta F''' + FF'' = 0$ with boundary conditions $F(0) = 0$, $F'(0) = 1$, $F'(\infty) = 0$ and $\eta = r^2(4z)^{-1}(\lambda Q/\mu^3)^{1/2}$. Tables are given for F and its derivatives for $\eta = 0(.1)3(.5)10(1)20(10)100$. Analysis is extended to a heat source in a vertical moving column of air with particular attention to limiting cases of large and small vertical velocities. N. A. Hall, USA

437. Shuler, K. E., Dames, C. A., and Laidler, K. J., The kinetics of membrane processes. III. The diffusion of various non-electrolytes through collodion membranes, *J. chem. Phys.* **17,** 860–865, Oct. 1949.

Comparisons with experiments are made. Good agreement between computed and observed diffusion rates is obtained. Computations of energies of activation and entropies of activation for solvent and solute are made, utilizing experimentally determined rates of pressure variation. M. Tribus, USA

438. Kronig, R., and Brink, J. C., On the theory of extraction from falling droplets, *Appl. sci. Res.* **A 2,** 2, 142–154, 1950.

A theory of solvent extraction based on diffusional theory has long been needed by chemical engineers. Article supplies the necessary approach to the problem by determining, from the motion of liquid spheres and mass-transfer considerations, the time required for a droplet to decrease the amount of soluble substance within it by a stipulated amount. Unfortunately, authors have not considered the problem of "skin" resistance associated with the diffusion process which governs the rate of extraction. No experimental data are presented to test the theory, although an example showing compliance with practical results is included. J. M. DallaValle, USA

439. Benfield, A. E., The temperature in an accreting medium with heat generation, *Quart. appl. Math.* **7,** 436–439, 1950.

A semi-infinite medium $x > 0$ moves with a constant velocity v in positive direction of x-axis, and material at the face is replenished at a constant rate so that this face maintains a fixed position $x = 0$. Initial temperature of medium and temperature of new material have same constant value; also all the material generates heat at a constant rate. Temperature $T(x, t)$ then satisfies a differential equation of the type $T_t = kT_{xx} - vT_x + a$, where k and a are constants. The boundary-value problem in $T(x, t)$ is solved here, using Laplace transforms, to obtain a simple formula for $T(x, t)$ in terms of error functions.
Courtesy of Mathematical Reviews R. V. Churchill, USA

440. Shvets, M. E., Heat transmission in the laminar boundary layer on a solid of rotation (in Russian), *Prikl. Mat. Mekh.* **14,** 102–104, 1950.

Earlier results of author [title source, **13,** 257–266, 1949; Rev **3,** 1544] are extended to obtain polynomial expressions for velocity and thermal boundary-layer gradients for laminar incompressible flow. Same method of successive approximation is used and results assume a general external velocity and temperature gradient. The somewhat more specialized data of Fage and Falkner [*Aeronaut. Res. Comm. Rep. and Memo.* no. 1408, 1931] is confirmed and consistency with data of Pohlhausen [*Z. angew. Math. Mech.* **1,** 28–42, 1921] for a plane is established. N. A. Hall, USA

441. Casci, Corrado, On the temperature distribution in the permanent state of a ring in media of diverse temperatures (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (**8**) 7, 1949, 219–224, 1950.

A ring placed in two surrounding media A and B, which are separated by a diametral plane of the ring, is assumed to be in the permanent state regarding its temperature. Assuming medium A at constant temperature θ_A, and region B at constant temperature θ_B, author finds temperature θ interior to ring. Analytically the problem is the solution θ of the differential equation $d^2\theta/ds^2 + \alpha(\theta - \theta^*) = 0$, $0 \leqq s \leqq 2\pi r$, where $\theta^* = \theta_A$ for $0 < s < \pi r$, $\theta^* = \theta_B$ for $\pi r < s < 2\pi r$, and r and α are constants.
Courtesy of Mathematical Reviews F. G. Dressel, USA

442. Casci, Corrado, On the temperature distribution in a rotating ring in media of diverse temperature (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (**8**) 7, 1949, 297–303, 1950.

A ring imbedded in two surrounding media A and B, which are separated by a diametral plane of the ring, is assumed to be rotating with a constant angular velocity ω. Problem of paper is to find the distribution of temperature θ of ring, given that region A is at constant temperature θ_A and region B is maintained at constant temperature θ_B; initial temperature θ_0 of ring is also assumed known. Author finds θ by method of separation of variables, where θ satisfies conditions $\partial^2\theta/\partial s^2 + \alpha(\theta - \theta^*) = \beta\partial\theta/\partial t$, $\theta(s, 0) = \theta_0(s)$. Here $\theta^* = \theta_A$ for $0 < s' < \pi r$, $\theta^* = \theta_B$ for $\pi r < s' < 2\pi r$, and s' is related to s and t by the relation $s' = s + \omega rt$. Coefficients α and β are assumed constant and r is radius of ring.
Courtesy of Mathematical Reviews F. G. Dressel, USA

443. Verschoor, H., and Schuit, G. C. A., Heat transfer to fluids flowing through a bed of granular solids, *Appl. sci. Res.* **A 2,** no. 2, 97–119, 1950.

Authors investigated experimentally case of a gas passing through a tube having an isothermal wall and containing a bed of granular solid. They give the following expression for ratio of effective thermal conductivity of bed to that of gas $\lambda_p/\lambda_g = 1.72(\lambda_s/\lambda_g)^{0.26} + 0.10(Sd_t)^{0.50}(\pi m/4\eta Sd_t^2)^{0.69}$ where λ_s denotes conductivity of material from which the granules are formed, S granule-surface area per unit volume of bed, d_t tube diameter, m total mass flow-rate of gas, and η gas viscosity. Expression was tested with air and hydrogen, with beds of 35–45% voidage and 5–10 length-diam ratio, and with Reynolds numbers of 0–1000. In reviewer's opinion, it is not valid for gases with Prandtl numbers differing much from 0.75. H. G. Elrod, Jr., USA

444. Erugin, N. P., A closed solution of a parabolic inhomogeneous boundary problem (in Russian), *Prikl. Mat. Mekh.* **14,** 215–217, 1950.

The one-dimensional heat equation $u_t = u_{yy}$ with boundary conditions $u(0, t) = \text{constant}$, $u(l, t) = 0$ $(t > 0)$, $u(y, 0) = 0$, $0 < y < l$, is solved in terms of an improper integral. Author selects a certain improper integral which satisfies the differential equation and the first two boundary conditions and expands it for $t = 0$ in a Fourier series in y and determines coefficients so that remaining condition is satisfied. This solution is combined with usual Fourier series solution to give solution in closed form. Same process may be applied if second boundary condition is modified to read $u(l, t) = \text{constant}$.
Courtesy of Mathematical Reviews C. G. Maple, USA

445. **Garavaldi, Orestina, On a problem of thermic propagation treated by the method of functional operators** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **6**, 461–466, 1949.

Problem referred to in title, and an analogous problem in electrical theory, lead to the solution $q(t) = \dfrac{\alpha\gamma}{\alpha\gamma + \coth\gamma^s} Q(t)$, where $Q(t)$ is a given function of time, α, s are constants, and γ^2 is a constant multiple of d/dt. Author first obtains the solution for case when $Q(t)$ is the integral of the unit function. Solution of general case follows by Duhamel's theorem. Whole process runs parallel to standard treatment of such problems [cf. for instance Carslaw and Jaeger, "Conduction of heat in solids," Oxford, 1947, chapters III, XI, in particular, §§ 41, 117] except that instead of theory of Laplace transformation, Giorgi's theory is used to justify the formal operations.

Courtesy of Mathematical Reviews A. Erdélyi, USA

446. **Eckert, E., and Lieblein, V., Transfer of matter from a plane surface into a flowing gas at large drop of partial pressure** (in German), *Forsch. Geb. Ingenieurw.* **16**, 2, 33–42, Sept./Oct. 1949.

If drop of partial pressure is not small, transfer of matter can no longer be derived from heat transfer but has to be calculated differently. Paper deals with transfer of matter from a plane surface into the laminar boundary layer of a gas flowing along it. Results are used for a detailed calculation of diffusion of vapor into a flow of air.

From authors' summary

Acoustics

447. **Martin, M. H., and Jackson, G. B., The sound waves generated by a particle at supersonic speed,** *Schweiz. Arch. angew. Wiss. Tech.* **16**, 114–119, 1950.

Authors determine the acoustic radiation pattern from a particle moving under action of a constant gravitational field and a resisting force depending only on velocity and having a subsonic limiting speed. It is shown that the family of waves simply covers all space, that a fixed observer will hear the particle only once, and that the apparent source is uniquely determined.

J. W. Miles, USA

448. **Price, W. J., and Huntington, H. B., Acoustical properties of anisotropic materials,** *J. acoust. Soc. Amer.* **22**, 1, 32–37, Jan. 1950.

Several acoustic properties of anisotropic media have been investigated by the pulsed ultrasonic method and results analyzed in terms of plane waves traveling through an infinite medium. Single crystals of ammonium dihydrogen phosphate and potassium dihydrogen phosphate were subjects of study and, in each case, velocity measurements were used to compute a complete set of elastic moduli. Coupling between modes, which results in simultaneous generation of two modes by a transducer and in transfer of energy between modes during a reflection, was studied. Phenomenon of acoustical birefringence in ADP was particularly investigated and it was found that all three modes with wave normals in the 45-deg direction in the yz plane are propagated in different directions, making nonzero angles with wave normals. A case in which anisotropic property of medium causes anomalous beam spreading is reported, and circumstances leading to this condition are discussed.

From authors' summary

449. **Pachner, Jaroslav, Pressure distribution in the acoustical field excited by a vibrating plate,** *J. acoust. Soc. Amer.* **21**, 617–625, Nov. 1949.

An equation is given for pressure distribution in the acoustical field excited by a vibrating circular plate clamped at the edge in an infinite wall for points, distance of which from center of plate is greater than 10 to 20 times its radius. The resultant pressure is obtained by a superposition of the partial pressures. Exact quantitative information is given for the partial pressures. However, if forces which set plate in motion and vibrational mode of plate are not known, any theoretical conclusions about resultant pressure distribution are of limited validity.

Robert O. Fehr, USA

450. **van Itterbeek, A., and Slootmakers, P., Measurements on the absorption of ultrasonics in water,** *Physica Hague* **15**, 897–905, Oct. 1949.

Measurements of attenuation of water show a value proportional to square of the frequency for frequencies above five megacycles when measured by pulse method, Debye-Sears optical method, and by balance method. For frequencies below this, data have been published indicating that attenuation increases over this value. Most of these measurements have been made by the acoustic balance method.

Paper examines sources of error for balance method and shows that errors can enter which increase indicated attenuation by factors as high as 10. Some of these errors are pressure on the crystal, crystal orientation, and cavitation in liquid. When these errors are evaluated, experimental values for attenuation are obtained at 1.5 megacycles that agree with other measurements at higher frequencies. Hence, there is no indication of a dispersion in the attenuation or velocity for water. W. P. Mason, USA

451. **Hazzard, George W., Ultrasonic absorption in liquids,** *J. acoust. Soc. Amer.* **22**, 29–32, Jan. 1950.

Effect of molecular symmetry on absorption of sound waves in liquids is studied for liquid pairs, benzene and toluene; paraxylene and orthoxylene; and cyclohexane and methyl cyclohexane. Absorption is measured by a Debye-Sears cell by comparing light intensity at two different points on wave path. Difference in absorption between benzene and toluene (which amounts to a factor of 30) has been associated with the symmetrical structure of benzene as compared with dissymetrical structure of toluene. Other pairs chosen had the same comparison of a symmetrical structure with a dissymetrical structure composed of same chemical elements. Measured results for the other pairs did not show a larger absorption for symmetrical structures than occurred for dissymetrical structures. Presence of single and double bonds in the two substances, tetrachlorethylene and tetrachloroethane, also does not seem to affect their relative absorption. Author also investigates effect of long chains of polystyrene in benzene and toluene. Although viscosity increases by large factors, the high frequency attenuation is unchanged. This lack of absorption in long chain molecules at high frequencies has recently been shown by reviewer to be due to relaxation of chain-segment motion at high frequencies. W. P. Mason, USA

452. **Dingle, R. B., The hydrodynamics of helium II,** *Proc. phys. Soc. Lond.* Sect. A, **62**, 648–655, Oct. 1949.

Experiments with liquid helium II have shown that, apart from the ordinary sound wave due to pressure fluctuations, there is also a second type of sound wave characterized by temperature fluctuations. On the supposition that fluid is a mixture of two phases, one the normal fluid, and the other the superfluid (assumed inviscid and to carry no entropy), apparently different

derivations of velocity of second sound have been given by Landau and Tisza. In this paper, the Lagrangian of system is used to derive force acting on superfluid alone, and on normal fluid alone, and, hence, to show that derivations of Tisza and Landau are equivalent even if thermal expansion is not neglected. [In first equation of paper read ρ for p. Third equation, which author describes as kinematical, is, in fact, equation of motion of an incompressible inviscid fluid under no forces.]

L. M. Milne-Thomson, England

453. Dutta, A. K., On the dispersion of supersonic vibrations in liquids, *David Taylor Model Basin Transl.* 147, 3 pp., Oct. 1949.

Paper is a translation of a work in German (1938), which deals with dispersion of sound waves in water. Using a Debye-Sears cell, velocity of water is found to increase by 0.15% in frequency range from 3×10^6 cycles to 3×10^7 cycles. However, other investigators have questioned validity of this result. In recent 5th edition of Bergmann's "Ultrasonics" it is stated: "These positive results, however, were simulated either by temperature effect or by deformations of the sound field; Matossi, for instance, has checked the very careful measurements of Dutta, who found an increase of velocity of sound by 0.15% in water, and demonstrated that they were simulated by local heatings of the sound field."

Warren P. Mason, USA

454. Goetz, Harald, A method of measuring radiation pressure in ultrasonic waves (in German), *Z. Naturforsch.* **A 4a**, 8, 587–588, Nov. 1949.

By releasing air bubbles in a horizontally directed ultrasonic field in a liquid, bubbles are caused to rise along a sloping path, owing to radiation pressure. Assuming a plane progressive wave, radiation pressure and, hence, intensity may be calculated from angle of rise, velocity of normal rise, and properties of liquid. Preliminary results agree in order of magnitude with those of other methods; a more refined analytical treatment is promised later.

Vincent Salmon, USA

455. Storruste, A., and Wergeland, H., On two complementary diffraction problems. I. Circular hole and disc in confocal coordinates, *Norske Vid. Selsk. Forh., Trondheim* **21**, 10, 38–42, 1948.

Storruste, A., and Wergeland, H., On two complementary diffraction problems. II. Transmission of sound through a circular hole, *Norske Vid. Selsk. Forh., Trondheim* **21**, 11, 43–48, 1948.

Authors are concerned with diffraction of a normally incident plane wave of sound by a circular disk or hole in an infinite plane screen. Both disk and screen are assumed to be rigid and infinitely thin. The wave equation is separated in bolate-spheroidal coordinates and the wave field expanded in spheroidal wave functions of order zero. A simple expression is derived for transmission coefficient of the circular hole. [Authors would like to make following corrections: (1) Paragraph relating to Casimir should be deleted (p. 39); (2) M-functions are finite at $\mu = 0$ (p. 45, lines 5 and 6 from bottom); (3) in the addendum (p. 47, line 3 from bottom) "these authors" refers to Levine and Schwinger.]

Courtesy of Mathematical Reviews C. J. Bouwkamp, Holland

456. van Itterbeek, A., and Verhaegen, L., Measurements of the velocity of sound in liquid argon and liquid methane, *Proc. phys. Soc. Lond.* Sect. B, 12, 800–804, Dec. 1949.

The ultrasonic velocity in liquid argon and methane has been measured between their boiling and melting points and the ratio of specific heats calculated. Compressibilities and degrees of association have also been calculated and compared with results obtained for other liquefied gases (hydrogen, helium).

From authors' summary

457. Glaubitz, Heinz, Noise measurements on gears (in German), *Arch. tech. Messen*, no. 158, V56-1, Dec. 1948.

458. Rohde, L., Kupfmuller, K., and Krause, A., Dynamometer for tone frequency (in German), *Arch. tech. Messen*, no. 159, J741–11, Dec. 1948.

459. Brillouin, Jacques, Direct proof of Fresnel's formula for the diffraction of a plane wave by a reflecting semi-plane (in French), *C. R. Acad. Sci. Paris* **229**, 513–514, 1949.

Diffraction of a monochromatic, plane wave of sound by a perfectly reflecting half plane is treated by the introduction of parabolic coordinates. [For a rather complete bibliography on this problem, see H. Bateman, "The mathematical analysis of electrical and optical wave motion on the basis of Maxwell's equation," Cambridge University Press, 1915, pp. 82–88.] The results are used to establish directly Fresnel's formulas for this particular problem.

J. W. Miles, USA

460. Kharkevich, A. A., A new method for solving diffraction problems (in Russian), *Doklady Akad. Nauk SSSR* (N.S.) **72**, 45–47, 1950.

Method is to reduce problem to a solved diffraction problem by a coordinate transformation. This is achieved in case of motion of a semi-infinite plane shock wave (zero pressure before wave front, unit pressure at all points behind it) over a rigid half-plane screen, diffraction starting when the shock front reaches the edge. This is reduced to the problem of a similar wave radiated normally away from the screen. Author thus confirms his previous work on the original problem [*Žurnal Teh. Fiz.* **19**, 828–832, 1949].

Courtesy of Mathematical Reviews F. V. Atkinson, USA

461. Bouwkamp, C. J., On the freely-vibrating circular disk and the diffraction by circular disks and apertures, *Physica* 16, 1–16, Jan. 1950.

Approximate solutions for problem of diffraction of normally-incident sound waves around circular disks and through circular holes have been obtained in the past, but they have yielded only the first few terms in the series which are required for a complete solution. Author gives a new approach to problem and includes the closely allied case of freely vibrating circular disk. For wavelengths large as compared with the radius of the disk or aperture (the case of ordinary acoustic waves), the final solution is presented in form of series with ascending powers of product of wave number and obstacle radius. Author suggests possibility that same method may prove applicable to case of diffraction of transverse electromagnetic waves.

Solution starts from a formulation of the problem in terms of an integrodifferential equation and some of the details of solution are omitted so that article can be brief. Paper includes a complete review of previous related work on subject, and final section of article considers the energy relations of the freely vibrating disk.

Wm. C. Johnson Jr., USA

Ballistics, Detonics (Explosions)

462. **Wimpress, R. N., Internal ballistics of solid-fuel rockets,** New York, McGraw-Hill Book Co., Inc., 1950, xii + 214 pp. $4.50.

Book is a compilation of material accumulated in the period 1941-1946 at California Institute of Technology, under government contract. After short mention of the connection between internal ballistics and performance (chapter 1), a brief and incomplete account is given (chapter 2) of thermal and chemical characteristics of propellant gases. Chapter 3 is devoted to burning properties of propellants. Chapters 4, 5, 6 contain derivation of equations of one-dimensional motion of gas for variable mass flow, and their application to study of rocket nozzles and of flow conditions inside rocket motors, together with their effect on nozzle performance and burning rate. Chapter 7 leads to computation of equilibrium pressure and of its dependence on geometry, temperature, and grain deformation. Chapters 8, 9, and 12 are devoted to design of different types of propellant grains burning on their complete area or with burning area control; additional information is given on multiple grain charge and on stabilization of irregular burning. Chapter 10 contains some discussions on ignition problems. Chapter 11 discusses effects of elastic behavior of grains during firing. Chapter 13 presents, briefly, the testing technique. Chapter 14 is devoted to heating of walls and propellant from burned gases and to nozzle erosion effects.

Material is discussed without reference to work from other sources, as explicitly stated in preface. As a result, treatment is inhomogeneous and incomplete both in theoretical and in experimental presentations. Nevertheless, book has a real utility in presenting information which was formerly available only to a restricted number of persons. Luigi Crocco, USA

463. **Polachek, Harry, Solution of the differential equations of motion of a projectile in a medium of quasi-Newtonian resistance,** *Quart. appl. Math.* **7**, 275-291, Oct. 1949.

Author investigates flight of a projectile in a vertical plane, first assuming a Newtonian air resistance law, $R = KV^2$. Classical solutions of Bernoulli and D'Alembert are given, and, by writing the variation equations, an estimate is obtained of effects of perturbed initial conditions, and of a force law which is perturbed from being Newtonian. To obtain terminal conditions of trajectory, system of equations adjoint to these variation equations is written and solved.

Method used is to first study case of perturbed initial conditions with a Newtonian force law, for which case the variation equations are homogeneous. Case of a perturbed force law is then treated by variation of constants. From the adjoint system, approximate formulas are obtained for error incurred in range and in time of flight due to assumed perturbations. Finally, simplified use of vertical trajectories, excluded from previous considerations, is treated. Felix Rosenthal, USA

464. **Athen, Hermann, Mathematical exterior ballistics (Mathematische Aussenballistik)**, *Naturforschung und Medizin in Deutschland 1939-1946*, Band **7**, pp. 121-169. Wiesbaden, Dieterich'sche Verlagsbuchhandlung, 1948. DM 10.

Author presents a condensed but equation-packed summary of progress of German exterior ballistics in indicated period. Integrability of the differential equations (for standard conditions) for particle theory is re-examined. Methods using integral equations were proposed. Results by E. Leimanis, extending work of I. Drach, deal with infinitesimal transformations not themselves involving the ballistic coefficient. Numerical, graphical, and instrumental methods are reviewed, with examination of bounds for errors. Transformations of basic equations render them more readily adapted to numerical methods. A simple approximate finite solution of theoretical equations by R. Schmidt was found to give surprisingly close approximations to those obtained by more elaborate numerical methods. Studies on entire family of integral curves for given common initial conditions are particularly useful. Author considers series expansion in terms of $\sin \theta_0$, and the form of terms through the quadratic. Oblique coordinates (z, r) $(r = x \tan \theta_0 - z)$ serve well to provide a number of useful equations and inequalities. Author examines accuracy of interpolation among pencil of complete trajectories, recommending quadratic rather than linear interpolation. Discussion of standard trajectories concludes with a critical study of an inverse problem, that of computing resistance function from observed firings. Author next reviews perturbation theory (for particle case). A fourth section is devoted to linearized theory of ballistic oscillations, incident to force and moment systems, upon a rotating projectile. Last section examines theory of supersonic resistance and determination of the associated optimal form of projectile. A. A. Bennett, USA

465. **Popov, B. S., On a Karamata condition for the integrability of the ballistic differential equation** (in Serbian, with French summary), *Filoz. Fak. Univ. Skopje* **2**, 249-263, 1949.

By means of the Legendre differential equation, and then by means of the hypergeometric differential equation, author proves the Karamata integrability condition of the differential equation of exterior ballistics:

$$(y + Ae^{nx} + Be^{-nx} + R)y^1 + y^2 - 1 = 0.$$

From author's summary

466. **Reinhardt, Thomas F., Factors affecting the range of rocket-powered aircraft,** *Aero. Engng. Rev.* **8**, 32-35, Oct. 1949.

Paper discusses relative importance of specific impulse and density impulse of rocket propellants and influence on performance of rocket-powered aircraft. Three significant characteristics of rocket power plants which differentiate them from air-consuming engines are stated: (1) Weight and volume of propellants are large in comparison to size of rocket engine. (2) Rocket thrust is independent of flight velocity and increases somewhat with increasing altitude. (3) Specific propellant consumption is several times as high as that of an air-consuming engine, and, based on present propellants, it does not appear as though consumption will ever decrease below 15 lb per hr per lb of thrust.

It is also pointed out that, contrary to conventional aircraft, weight of rocket-powered winged aircraft is of minor importance compared with the volume. Consequently, specific propellant consumption on a volume basis is more significant than on a weight basis. It is further shown that range of a rocket plane is directly proportional to density impulse of propellant for an aircraft with drag essentially independent of weight. Density impulse is the product of specific impulse and density. For representative propellants, density impulse will fall in range between 15,000 to 20,000 lb sec per cu ft.

Paper concludes that rocket-powered aircraft will be small in size but will have large weight and high wing loading. It will be a short range craft with exceptionally high take-off, climb, and flight speeds. J. T. Rettaliata, USA

467. **Bolz, Ray E., A note on the approximate plane motion during the burning period of rocket-propelled missiles launched at small angles of yaw from aircraft,** *J. aero. Sci.* **17**, 114-120, 1950.

Paper contains a solution of the equations of motion of a rocket

which is valid under following assumptions: (1) Gravity is neglected. (2) Coefficients of cross forces and cross couples are constant. (3) Rocket's thrust is constant and large in comparison with drag forces. (4) Change of mass of missile during burning period can be neglected. (5) Angle of yaw α is small enough to permit the approximations $\sin \alpha = \alpha$, $\cos \alpha = 1$. (6) Missile has no yawing angular velocity at launch. (7) Rocket is axisymmetrical both in design and performance. In view of these, the assumption that rocket is fired from an aircraft affects the initial conditions only. Solutions obtained are of usual type, being, for example, particular cases of those obtained by reviewer [*Philos. Trans. roy. Soc. Lond. Ser. A* **241**, 457–585, 1949, § 5; Rev **3**, 377]. Their value lies in the fact that they are expressed in terms of three dimensionless parameters, so that, with aid of the series of graphs given, it should be easy to apply them to a number of missiles of different design.

One minor point, which affects the numerical value of one of author's parameters but not his general theory, seems worth mentioning. There is very little evidence, theoretical or experimental, for assuming that coefficients of cross-spin force and cross-velocity force are connected by the relation which he postulates, and it is only for missiles possessing lifting surfaces of relatively large area that this relation is likely to hold approximately. [See footnote on p. 480 of paper cited, or J. B. Rosser, R. R. Newton and G. L. Gross, "Mathematical theory of rocket flight," New York, McGraw-Hill, 1947, pp. 29–32; Rev **1**, 912].

Courtesy of Mathematical Reviews R. A. Rankin, USA

468. Taylor, G., The dynamics of the combustion products behind plane and spherical detonation fronts in explosives, *Proc. roy. Soc. Lond. Ser. A* **200**, 1061, 235–247, Jan. 1950.

Author shows that, after initial disturbances have died out, a plane detonation wave consists in a centered rarefaction wave, which may be complete or joined to a uniform region at rest, depending on the rear boundary condition. This follows since the front is a characteristic and entropy is uniform behind it. The whole picture expands steadily with time.

A steadily expanding solution is then found for a spherical detonation wave. A uniform region extends from the center to about half way to the front and joins on to a rarefaction wave of more complicated type. Since front is now an envelope of characteristics, the pressure and velocity gradients behind it are infinite. Numerical results, based on published thermodynamic data for TNT, are given for both cases.

H. H. M. Pike, England

Soil Mechanics, Seepage

469. Verschoor, H., Experimental data on the viscous force exerted by a flowing fluid on a dense swarm of particles. *Appl. sci. Res.* **A 2**, 2, 155–161, 1950.

Experiments on upward flow through a porous bed are described for various stages of suspension. Verification is found for a modification of Darcy's law presented by Brinkman, in which the viscous force is expressed in terms of ratio of volume of expanded bed to volume of solid material.

John S. McNown, USA

470. U. S. Army, Corps of Engineers, Relief well systems for dams and levees on pervious foundations, *Wways. Exp. Sta., tech. Memo.* 3-304, 104 pp., Nov. 1949.

This memorandum describes results of sand model studies on behavior of relief wells intended for control of underseepage and sand boils on the landside of dams and levees built on a pervious foundation. Investigation involved study of effects of well spacing and penetration, seepage entrance conditions, characteristics of landside top stratum and depth, permeability and stratification of pervious foundation stratum, which in all cases was underlain by an impervious stratum.

Principal conclusions reached are: (a) Relief wells of proper spacing and penetration will reduce effectively the excess hydrostatic pressure on landside of levees; (b) presence of wells will somewhat increase the total flow, but seepage outside of wells may be reduced materially; (c) it is important that wells should penetrate main water-carrying strata; and (d) with regard to well flows and landside pressures, a close agreement was found between theoretical formulas and test results, when conditions assumed in formulas were fulfilled. A summary of existing formulas pertaining to relief wells is given in the appendix.

Alexander Hrennikoff, Canada

471. Nelson-Skornyakov, F. B., Filtration in a homogeneous medium (Filtratsiya odnorodnoĭ srede) (in Russian), *Izd. Sovetskaya Nauka*, 1949, 558 pp.

472. Hilf, J. W., Estimating construction pore pressures in rolled earth dams, *Proc. sec. int. Conf. Soil Mech. Found. Engng.* **3**, 234–240, 1948.

Natural soils consist of solid particles and voids that are filled with water and air. If a static load is applied by means of a tight-fitting piston, permitting neither air nor water to escape, it is found that there is a measurable volume reduction of the soil mass. Final pressure associated with air in the voids can be computed by combining Boyle's law with Henry's law for solubility of air in water, and author writes the equation between pore pressure p and consolidation Δ. This equation can be plotted in diagrams for laboratory samples. But if consolidation occurs without permitting drainage, the induced pore pressure will always be smaller than conventional total stress σ, experiments indicating that $\sigma = \bar{\sigma} + p$ ($\bar{\sigma}$ is effective normal stress). When drainage is permitted, a relation can be deduced between $\bar{\sigma}$ and Δ in soils for condition of complete lateral restraint. From p vs. Δ and $\bar{\sigma}$ vs. Δ curves, the σ vs. Δ and $\bar{\sigma}$ vs. p curves can be plotted. Nomographs have been devised which solve the necessary equations and facilitate a statistical treatment of the problem.

Field measurements (from piezometers for pore pressure and from telescoping pipe-crossarm apparatus for vertical consolidation) show that pore pressures are developed in the impervious zone of a rolled earth dam during construction, coincident with consolidation of the fill under its own weight. Curves deduced from the field measurements seem to indicate that if volume change characteristics of a soil can be established in the laboratory, a reasonable estimate of pore pressure can be made from compaction properties of the material. L. J. Tison, Belgium

473. Averyanov, S. F., Approximate estimation of the role of seepage in the capillary zone (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **69**, 3, 309–312, Nov. 1949.

Paper presents derivation of an approximate expression for seepage discharge in capillary zone, assuming capillary flow taking place parallel to main flow and under same gradient. Coefficient of permeability is assumed to vary in a suitable manner through capillary depth as a power function of water content, which itself is taken as a power function of depth. The simple expression obtained seems to agree reasonably well with laboratory experiments. Alexander Hrennikoff, Canada

474. **Göhre, K., On the passage of seepage water** (in German), *Z. Met.* **3**, 352–356, Nov.-Dec. 1949.

Results of measurements on downward flow of water through sandy soils are reported. Tests were performed under as nearly natural conditions as possible. Cast iron boxes 1.5 meters deep with soil in question were placed out of doors on scales where they received the naturally occurring precipitation. A layer of gravel was placed in the bottom of the boxes and arrangement made for drawing off and measuring any water collected. Daily records were kept of precipitation, amount of percolating water collected, and weight of boxes. Maximum rate of flowing into the collecting basin following a rain would occur from one to four days after rain, depending on amount of precipitation and condition of the soil. Records were kept for the period of 1930 to 1937.

Gerald Pickett, USA

475. **Moore, R. Woodward, Geophysical methods of subsurface exploration in highway construction,** *Publ. Roads* **26**, 3, 49–64, Aug. 1950.

The Bureau of Public Roads has designed and built portable equipment for two types of tests, which were described in *Public Roads* in 1935. Subsequent work, carried on during past 15 years in 21 states, has established both methods as most useful, rapid, and economical means of obtaining preliminary information on depth and nature of subsurface formations. In this article are presented a review of the theory and method of operation of the two types of equipment, and results of a number of actual field surveys made with them.

It is evident from these data that, while both geophysical exploration methods are useful, the earth-resistivity test is more universally applicable to a variety of highway construction problems than the refraction seismic test. Detailed subsurface surveys can best be made by initial use of the resistivity equipment, followed by check tests with the seismic apparatus where needed.

From author's summary

476. **Shepard, E. Raymond, Subsurface explorations by geophysical methods,** *Proc. Amer. Soc. Test. Mat.* **49**, 993–1015, 1949.

Principles of both seismic and electrical resistivity methods of subsurface exploration are described and illustrated. Results of field tests, as made by the Corps of Engineers, show the methods to be relatively rapid and inexpensive in determining depth of overburden as well as some of its physical properties. Seismic method has been particularly useful in study of large dam sites, while electrical resistivity method is more applicable to highway and levee construction. Methods of interpretation and analysis are described. (From author's summary by J. C. G.)

R. W. Moore, in discussion, described use of resistivity method by Public Roads Administration. Method is of real value in locating deposits of sand and gravel, in classifying soils, and in studying bridge foundations. M. N. Sinacori lists uses of methods by N. Y. State Department of Public Works. Geophysical exploration of overburden is most valuable when used in conjunction with usual boring methods to reduce time and expense when a considerable amount of work must be done.

John C. Geyer, USA

477. **Jelinek, Richard, The force propagation in a semi-space for lateral-isotropic bodies** (in German), *Abh. Bodenmech. Grundl.*, 28–33, 1948.

For stratified soils, Young's modulus being different in horizontal and vertical directions (E_1 and E_2), author generalizes the classical theory of elasticity for lateral isotropic media and hopes to obtain stress formulas giving results closer to reality in foundation problems.

Starting from equations for equilibrium, he makes use of correspondence between related functions for plane stress, and gives stress function for the semispace acted upon by concentrated force. Unknown functions and constants are determined by compatibility and boundary values. Stresses are given as functions of $k^2 = E_1/E_2$. Integrating σ_z of the spatial problem, he obtains the corresponding plane strain and proves that stress formulas for case of plane stress and plane strain differ only in values of the elastic constants in the generalized case ($k^2 \neq 1$) as well. Regarding uncertainty in determining elastic constants experimentally, he deals in detail with case $m = \infty$ and proves that, according to experimental results, stresses concentrate around the force axis. The same effect being exerted by frictional stresses on base of footings and Young's modulus increasing with depth, author proposes to take these effects into account by using his equations with a suitable adjustment of constant k^2, thus avoiding cumbersome formulas derived for above cases.

A. Kézdi, Hungary

478. **Keverling Buisman, A. S., Computation of a quay wall,** *Proc. sec. Int. Conf. Soil Mech. Found. Engng.* **2**, 91–95, 1948.

Author presents a method of design for a quay wall in which wall rests on a slab foundation which, in turn, is carried by piles. Expressions are derived for computing state of stress at any point in soil foundation, taking into account such influencing factors as pile loads, pile deflections, and soil constants.

Woodland G. Shockley, USA

479. **Averyanov, S. F., Relation between water permeability of soil-grounds and their air content** (in Russian), *Doklady Akad. Nauk SSSR* **69**, 141–144, Nov. 1949.

Two values of coefficient of permeability are considered, one for saturated flow, the other for nonsaturated flow. Starting from expression of velocity of flow in a cylindrical tube and considering that a part of the soil pore moisture attracted by walls of capillary does not participate in flow process, it is found that discharge through a capillary in case of a nonsaturated flow (Q_ω) equals μQ_0, where Q_0 is discharge computed using the Poiseuille formula. In the same way μ is ratio of the coefficients of permeability for saturated and nonsaturated flow. μ is expressed as $\mu \times w^n$ where w is the "relative" moisture content and $n = 3.5$ (approximately).

Gardner's statement [*Soil Sci.* **11**, 1921] about the proportionality of the capillary permeability and moisture content is challenged. Results of computations are compared with corresponding data by Moore [*Hilgardia* **12**, 6, 1939] and Richards [*J. Amer. Soc. Agr.* **28**, 1936].

D. P. Krynine, USA

480. **Schiff, Leonard, and Dreibelbis, F. R., Preliminary studies on soil permeability and its application,** *Trans. Amer. geophys. Un.* **30**, 759–766, Oct. 1949.

Permeability of soil cores of Keene silt loam and Muskingum silt loam were studied in laboratory by determining velocity of water movement through core samples (transmission rate) and volume of water passing through sample when moisture content was such that amount of water entering was the same as that leaving (percolation rate). Both rates were plotted against soil moisture content at time of field sampling and an attempt was made to correlate the results with field data from natural watersheds. Data obtained showed extreme scattering, indicating that a very large number of tests would have to be run to yield results which might be of value in estimating potential infiltration rates and surface runoff.

E. Vey, USA

481. Lee, Donovan H., and Riches, C. E., The location of anchor blocks in sheet pile and similar walls, *Civ. Engng. Lond.* **44,** 520, 576–577, Oct. 1949.

Model experiments were run to determine the minimum distance back from bulkheads to place anchor blocks. Details are shown for the apparatus with provisions to minimize arching. Required corrections are discussed. Observations are plotted for the horizontal pull developed on the middle of three blocks 5 in. apart at depths of 1, 2, and 3 in. and up to 19 in. from the wall. Results indicate that, if the foot of wall cannot displace, it is unduly conservative to require anchors to be beyond angle of repose, and it is satisfactory to require that a passive wedge in front of anchors shall not intersect active wedge behind wall.

Edward S. Barber, USA

482. Browzin, B. S., Approximate computation of anchored bulkheads, *Proc. sec. Inter. Conf. Soil Mech. Found. Eng.* **7,** 133–135, 1948.

483. Fröhlich, O. K., Stability against slipping of an earth mass on a circular cylindrical sliding plane with consideration of the stress distribution on this surface (in German), *Federhofer-Girkmann-Festschrift,* Wien, Franz Deuticke, 181–198, 1950.

Stability calculations are usually made by comparing moment of active forces with moment of reactive normal and shearing stresses along the shearing plane, which are presumed to be evenly spread. Writer investigates the influence of distribution of normal stress on the shearing plane by assuming some typical stress distributions, all of them being symmetrical about the center of sliding plane. It is found that, with different stress-distributions, the safety against sliding varies only about 15 per cent.

F. C. de Nie, Holland

484. Palmer, L. A., and Thompson, James B., The earth pressure and deflection along the embedded lengths of piles subjected to lateral thrust, *Proc. sec. Int. Conf. Soil Mech. Found. Eng.* **5,** 156–161, 1948.

Problem of computing lateral earth pressures, deflections, shears, and moments at all points throughout embedded lengths of either individual piles or a row of sheet piling is solved by difference equations. Analysis involves solution of 13 simultaneous difference equations, four of which apply to boundary conditions for problem and nine for lateral earth pressures at nine equally spaced points along embedded lengths of piles.

Illustrative numerical examples are presented. Method of differences permits use of various expressions for the variation in modulus of earth reaction k with depth of soil. Graphs and diagrams show deflection curve, load, moment, and shear curves for bent pile.

From authors' summary

485. Van Zelst, Theodore W., An investigation of the factors affecting laboratory consolidation of clay, *Proc. sec. Int. Conf. Soil Mech. Found. Eng.* **7,** 52–61, 1948.

Test results indicate that the sample disturbance developed during preparation of specimens for testing has a serious effect on accuracy of data. The amount of disturbance (by volume) varied only slightly with specimen height. The preparation operation of planing top and bottom faces of a consolidation specimen was greatest source of disturbance in laboratory. Circumferential trimming disturbances were relatively small when compared to those developed in planing operation. In general, sample disturbance influences consolidation test results to a greater comparative degree than other factors such as friction and swelling. The thin specimen pressure-void ratio curves deviate widely from curves for thicker specimens in initial compression portions of the diagrams. The thick and thin curves are parallel but offset from one another in the virgin compression portions. These differences were found to be caused by sample preparation disturbances.

From author's summary

486. Da Silveira, Icarahy, On consolidation of an underplaced clay layer, support of statically indeterminate structures, *Proc. sec. Int. Conf. Soil Mech. Found. Engng.* **7,** 12–14, June 1948.

Author attempts to adjust results for settlement of a clay layer, obtained by classical two-dimensional consolidation theory of Terzaghi and Fröhlich, by considering change in consolidation load due to elastic deformation of structure under differential settlements. The expression for σ_z, the total consolidation pressure on layer is then written as $\sigma_z = (P - \zeta\sigma_i)/\omega$, where P is applied load, ζ elastic constant of structure, σ_i elastic deflection of structure, and ω surface area of layer.

Eben Vey, USA

487. Bernhard, R. K., Study on mechanical oscillators, *Proc. Amer. Soc. Test. Mat.* **49,** 1076–1036, 1949.

A mechanical oscillator to excite sinusoidal one- and two-dimensional vibrations has been developed which allows a continuous change of the resultant force vector in frequency, magnitude, direction, and action line while oscillator is in operation. Analytical and graphical solutions are presented. Construction of such an oscillator, including an automatic electromechanical control is described and some fields of application are discussed.

From author's summary

488. Gran Olsson, R., Rigorous solution of a differential equation in soil mechanics, *Quart. appl. Math.* **7,** 338–342, 1949.

An examination of the indeterminate form assumed at the initial value by the explicit solution of a specific linear equation of the first order.

Courtesy of Mathematical Reviews P. Franklin, USA

Geophysics, Meteorology, Oceanography

489. Berson, F. A., Summary of a theoretical investigation into the factors controlling the instability of long waves in zonal currents, *Tellus* **1,** 44–52, Nov. 1949.

Recent investigations dealing with long wave disturbances in horizontally uniform, barocline zonal currents show the important role of static stability and baroclinity. Here, an attempt is made to find an improved stability criterion by taking polytropic change of state into account. It is found that instability depends on Richardson's number R (= statics tability divided by square of vertical shear of wind), as in case of small-scale turbulence. Critical (minimum) number of waves around circumference is proportional to $(R)^{1/2}$. Thus, the often observed breaking up of zonal motion into a pattern of cyclonic and anticyclonic vortexes may be interpreted as transition of large-scale laminar motion into large-scale turbulent motion. Under average conditions only very long waves are found to be stable in middle latitudes. Results are derived under the rather restrictive assumption that all the energy necessary for growth of amplitude to occur is drawn from potential energy of system without changing kinetic energy of basic zonal flow. In case of strong westerlies in middle latitudes of Northern hemisphere, calculations from this theory are in reasonably good agreement with observational data. A more detailed report is to be published in *Communications of Sveriges Meteorologiska och Hydrologiska Institut,* series B.

Horst Merbt, Germany

490. **Kano, Kyoichi, On Richardson's criterion concerning the increase or decrease of turbulent energy in the free atmosphere** (in Japanese), *J. met. Soc. Japan* **27**, 10, 311–318, Oct. 1949.

From physical considerations, L. F. Richardson obtained an integral equation for change of kinetic energy of turbulence, from which he introduced a criterion for increase or decrease of turbulent energy in the atmosphere. Author shows that it is possible to derive the general expression for change of turbulent energy starting from fundamental equations of motion and making use of hypothesis of momentum transfer and concept of mixing length. Obtained result includes Richardson's criterion and is similar to the modified expression due to Rossby. Equation becomes linear when eddy viscosity is given as a function of height.

Itiro Tani, Japan

491. **Sutton, O. G., The application to micrometeorology of the theory of turbulent flow over rough surfaces,** *Quart. J. roy. met. Soc.* 75, 335–350, Oct. 1949.

A theory of atmospheric diffusion for smooth flow in conditions of neutral equilibrium is extended to rough flow through utilization of a new parameter. The parameter, macroviscosity, is defined as product of friction velocity and roughness length. Extended diffusion equation is given for two and three dimensions and is shown to compare favorably with observed data. Theories of rough and smooth flow and implications of introducing macroviscosity are discussed. It is concluded that formulation due to Rossby and Montgomery, in which influence of surface irregularities dies out with height, is treatment most appropriate for meteorology.

Philip Donely, USA

492. **Blue, Frank L., Jr. and Johnson, J. W., Diffraction of water waves passing through a breakwater gap,** *Trans. Amer. geophys. Un.* **30**, 705–718, Oct. 1949.

Experiments made in two water tanks, one 6 × 12 × 1 ft, the other 64 × 65 × 2 ft, are described. Diffraction coefficient (giving amplitude) and phase were measured at various points behind a barrier with a gap in it through which waves penetrated. In some experiments wave splitters, which essentially make the wave issue from a canal at right angles to barrier, were used. Most interesting phenomenon was appearance of breaks in wave fronts where amplitude was very small and phase changed along front. This is ascribed by authors to increase in velocity of waves with amplitude so that crest lines become unusually curved in places, but no definite trend in nature of breaks was noticed when amplitude was varied. Neither vorticity produced at ends of gap (which some previous workers had admittedly discounted) nor possible reflections at wave generator are considered.

An ingenious theoretical solution is proposed which, though not exact, is a good approximation; but it does not show the wave front breaks. It is based on the exact solution which had previously been obtained for a semi-infinite barrier and which is essentially Sommerfeld's classical solution for diffraction of light. It supposes sinusoidal waves of small amplitude, but authors do not seem aware that, for it to apply, the oscillations must also be simple harmonic and of one pure frequency, since water is a very dispersive medium (velocity of light in vacuo is independent of frequency), and we are not told how well the wave machine satisfied this condition.

The degree of approximation of a simplified solution, more appropriate when wave splitters are used, is not assessed; and it may be that breaks in wave fronts are not accounted for in the theory because it is not exact. It is noted that the simplified solution shows a tendency toward the breaks which appear more pronounced when wave splitters are used. R. S. Scorer, England

493. **Folsom, R. G., Measurement of ocean waves,** *Trans. Amer. geophys. Un.* **30**, 691–699, Oct. 1949.

Development of ocean-wave recorders during last few years is reviewed. Two shore wave recorders and a deep-sea wave recorder developed at the University of California are described. Ratio of pressure amplitude near surface to that near bottom derived from the linear theory is too low by 10%, according to laboratory test, and too low by 20%, according to tests at sea.

Walter H. Munk, USA

494. **Ursell, F., On the theoretical form of ocean swell on a rotating earth,** *Monthly Notices Roy. Astr. Soc. Geophys. Suppl.* **6**, 1–8, 1950.

In discussion of ocean swell, the waves studied by Gerstner (1802) and Stokes (1847) have received particular attention. One of the fundamental assumptions underlying these theories is that wave motion occurs on a nonrotating earth. Author shows that if curvature of earth can be neglected, the effect of earth's rotation makes swell waves in their stationary state differ very little from Gerstner waves. Work suggests that in general case each particle moves approximately in a horizontal circle of inertia as well as in the nearly vertical Gerstner motion. Diameter of inertia circles is greatest at surface, where it may be several hundred meters, and the period of revolution is about 12 csc l hours, where l is latitude. Except for this circular movement there is no mass transport, and thus there is no ground for supposing that ocean drift currents are due to fluid transport with the waves. On the other hand, waves appear to provide a reasonable mechanism for the generation of inertia currents which have been frequently observed. L. M. Milne-Thomson, England

495. **Berlyand, M. E., On the variation in time of the temperature in the surface layer of the atmosphere and the transformation of the mass of the atmosphere** (in Russian), *Doklady Akad. Nauk SSSR* (*N.S.*) **67**, 1017–1020, 1949.

The boundary-value problem considered is the following: to determine functions $T(z, t)$ and $\theta(z, t)$ satisfying system

$$\partial T/\partial t = (\partial/\partial z)[k(z)\partial T/\partial z] + Q(z,t), \quad 0 \leqq z < \infty$$
$$\partial \theta/\partial t = n\partial^2\theta/\partial z^2, \quad -\infty < z \leqq 0$$

where $k(z) = n_0 + k_1 z$ for $0 \leqq z \leqq h$, and $k(z) = n_0 + k_1 h$ for $z \geqq h$ (the numbers n, n_0, k_1, h being physical constants), and subject to conditions

$$T(0, t) = \theta(0, t), \quad -\lambda_0 T_z(0, t) + \lambda\theta_z(0, t) = B(t)$$

plus continuity of T and T_z across the height $z = h$. The numbers λ_0 and λ are physical constants and the functions $Q(z,t)$, $B(t)$, $T(z, 0)$ and $\theta(z, 0)$ are given functions. The Laplace transform method is employed and T is finally obtained as a finite sum of convolutions of known functions. A numerical example is given.

Courtesy of Mathematical Reviews J. B. Diaz, USA

496. **Bowden, K. F., and Proudman, J., Observations on the turbulent fluctuations of a tidal current,** *Proc. roy. Soc. Lond. Ser. A* **199**, 1058, 311–327, Nov. 1949.

Measurement of tidal currents in Mersey estuary by means of a current meter have revealed fluctuations of short period (of the order of a few seconds) and long period (of the order of one-half minute to several minutes). Amplitudes, periods, and autocorrelation of the short-period fluctuations give evidence of being associated with turbulence due to bottom friction.

Walter H. Munk, USA

497. Lineikin, P. S., Theory of convection in a rectangular basin (in Russian), *Izvestiya Akad. Nauk SSSR. Ser. Geograf. Geofiz.* **13**, 393–408, 1949.

Contribution to a mathematical theory which might explain the monsoonal air currents from oceans to continents. The model of problem considered is a rectangular basin filled with a liquid of uniform density. Walls and top of basin are insulated against heat escape. Bottom consists of two equal rectangular plates made of different materials. One of these plates is heated, while other is kept at a constant temperature. Circulation currents within liquid, which arise during experiment, are to simulate processes of atmospheric convection peculiar to a monsoon. The theory treats case after steady heat flow has set in. The velocity potential of liquid is found as solution within a rectangular region of a nonhomogeneous biharmonic equation. Solution is found by method of successive approximations. Numerical computations are made, and the lines of flow are plotted for a particular case. Maximum and minimum horizontal velocity components are found and located for his particular case.

Courtesy of Mathematical Reviews H. P. Thielman, USA

Lubrication; Bearings; Wear

498. Vogelpohl, G., On the foam formation in lubricating oils produced by cavitation (in German), *Forsch. Geb. Ingenieurwesens (B)*, **16**, 4, 109–112, 1949–1950.

499. Fogg, A., The length-diameter ratio of journal bearings and its relation to friction loss, *Proc. seventh int. Congr. Appl. Mech.* **4**, 180–189, 1948.

Dependence of the coefficient of friction μ upon ZN/P and the length-diameter ratio L/D is determined by experiment. Temperature dependence of viscosity is taken into account by measuring the running temperature.

Author mentions two types of safety criteria for bearings: First, if running conditions (journal speed, load intensity, or temperature) are changed, safety is characterized by distance between operating point and failure point along μ vs. ZN/P curve. This is nearly equal for various L/D conditions. Second safety criterion is for passage of extraneous particles in lubricant; this improves as L/D increases, due to lower bearing eccentricity.

To provide a design guide for choice of L/D, author determined friction forces as a function of L/D for various total loads. Two cases with different cooling were investigated. In both cases it is found that minimum friction is achieved for $L/D = 0.5$.

Ulrich Rost, Germany

Marine Engineering Problems

(See also Revs. 91, 195)

500. Archer, S., Screwshaft casualties—The influence of torsional vibration and propeller immersion, *Tr. Inst. mar. Eng.* **62**, 1, 43–84, Jan. 1950.

Article discusses in detail the tail-shaft failure of the 2500 ihp 76 rpm, three-cylinder, triple-expansion engine-propelled liberty ships. The failures are attributed to following: (1) A peak stress in order of **10,000** psi nominal when hitting a second order critical speed at 120 rpm, when, under ballast conditions in rough weather, propeller comes out of water and engine races. (2) A third-order critical speed at 80 rpm giving a nominal stress of approximately 4000 psi. (3) Presence of corrosion fatigue due to inadequacy of shaft sealing in area of propeller keyway where there is already high stress concentration. An interesting appendix covers possible error in torque, when there is strong propeller damping, which may be obtained by checking amplitudes of vibration at two points on a lineshaft without regard to their phase. Paper and discussions are presented in detail and should be read by all who are interested in marine shafting and vibration.

F. E. Reed, USA

501. Chardome, P., Propulsion by the Kort-nozzle (in French), *Tech. Wet. Tijdschr.* **18**, anniv. issue, 161–168, Oct. 1949.

Engineering development of ducted ships' propellers, model test results from towing tests at Wageninger tank, and operating experiences with some 1000 tugs thus far equipped with Kort nozzles, foremost of ducted propeller designs, have demonstrated that large increases of specific thrust can be achieved for vessels operating at low speeds.

Basic principle of Kort nozzle, by which increase of specific propeller thrust is achieved through placing a stationary shrouding around propeller has led to most spectacular results in applications to harbor and river tugs. Static thrusts were increased by more than 50% compared with conventional propellers with correspondingly high increases at low speeds. Early deficiencies in maneuverability of Kort-nozzle vessels have been remedied.

Increase of specific thrust which makes possible use of propellers of smaller diameter has improved performance of river tugs and self-propelled river vessels, especially in the uphill run on rivers of low depth and of appreciable slope. Kort nozzles, either single or multiple, are giving service of higher efficiency and at less cost of upkeep than paddle wheels, which they replace in depth of water as low as 4 ft. Ducted propellers are found to have a larger range of operation at high thrust than controllable-pitch propellers which are superior only within $\pm 20\%$ of design speed.

John R. Weske, USA

502. Vedeler, Georg, A Mathieu equation for ships rolling among waves, *Norske Vid. Selsk. Forth, Trondheim* **22**, 5, 113–123, 1950.

With a more exact treatment of the hydrostatic forces than is customary, the equation for the rolling motion of a ship in waves long compared with its beam leads to a differential equation of Mathieu's type. Author illustrates an approximate method of solution based in part on a paper of Brillouin [*Quart. Appl. Math.* **6**, 167–178, 1948] and, for a special choice of parameters, compares this approximate solution with two other possible ones. In deriving equations of motion, hydrodynamic forces are not considered and ship is taken to be of infinite length.

J. V. Wehausen, USA

503. Standard nomenclature and symbols for hull and propeller resistance and propulsion strength and vibration, *Brit. Shipbuild. Res. Assoc.*, Rep. no. 6, 8 pp., 1949.

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